ADVANCED INTERDISCIPLINARY DATA ASSIMILATION: FILTERING AND SMOOTHING VIA ESSE

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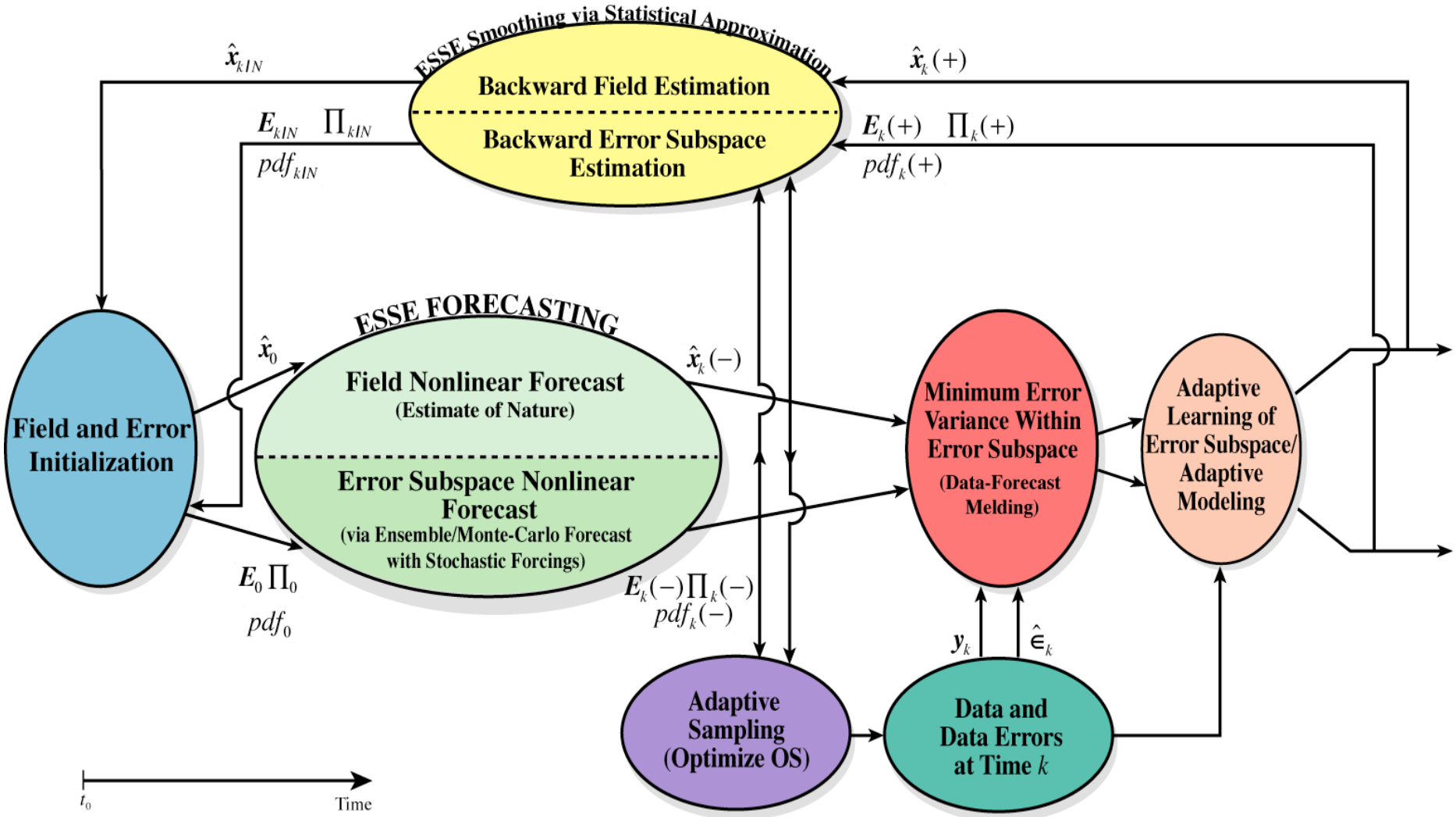
1. **ERROR SUBSPACE STATISTICAL ESTIMATION (ESSE)**
2. **BIOGEOCHEMICAL-PHYSICAL SMOOTHING IN MASSACHUSETTS BAY**



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DATA ASSIMILATION VIA ESSE

Table 1. Filtering/Smoothing via ESSE: Continuous-Discrete Problem Statement

Dynamical Model:	$d\widehat{\mathbf{x}} = \mathcal{M}(\widehat{\mathbf{x}})\,dt + d\widehat{\boldsymbol{\eta}}$, with $\widehat{\mathbf{x}}(\mathbf{r}_0, t_0) = \widehat{\mathbf{x}}_0$.
Measurement Model:	$\mathbf{y}_k^o = \mathcal{H}(\mathbf{x}_k) + \widehat{\epsilon}_k$.
Estimation Criterion:	
Estimate Error Subpace:	$\left\{ \text{Find } \mathbf{P}_k^p = \mathbf{E}_k \boldsymbol{\Pi}_k \mathbf{E}_k^T \text{ with } \text{rank}(\mathbf{E}_k) = p \;\middle|\; \min_{\boldsymbol{\Pi}_k, \mathbf{E}_k} \|\mathbf{P}_k - \mathbf{P}_k^p\| \right\}$
Estimate State by Min. Err. Var. in ES:	$\left\{ \text{Find } \widehat{\mathbf{x}}_k \;\middle|\; \min_{\widehat{\mathbf{x}}_k} J_k = \text{tr}\,[\mathbf{P}_k^p(+)] \text{ using } [\mathbf{y}_0^o, ..., \mathbf{y}_k^o / \mathbf{y}_N^o] \right\}$

- **Optimal error reduction and Min. Err. Var. combined:**
  - **"Determine the ocean state evolution by minimizing the most energetic errors, in agreement with the full dynamical model and measurement model (data) constraints, and their respective uncertainties."**

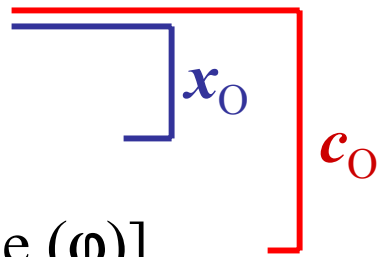
Coupled Interdisciplinary Error Covariances

$$\boldsymbol{x} = [\boldsymbol{x}_A \;\; \boldsymbol{x}_O \;\; \boldsymbol{x}_B]$$

Physics: $\boldsymbol{x}_O = [T, S, U, V, W]$

Biology: $\boldsymbol{x}_B = [N_i, P_i, Z_i, B_i, D_i, C_i]$

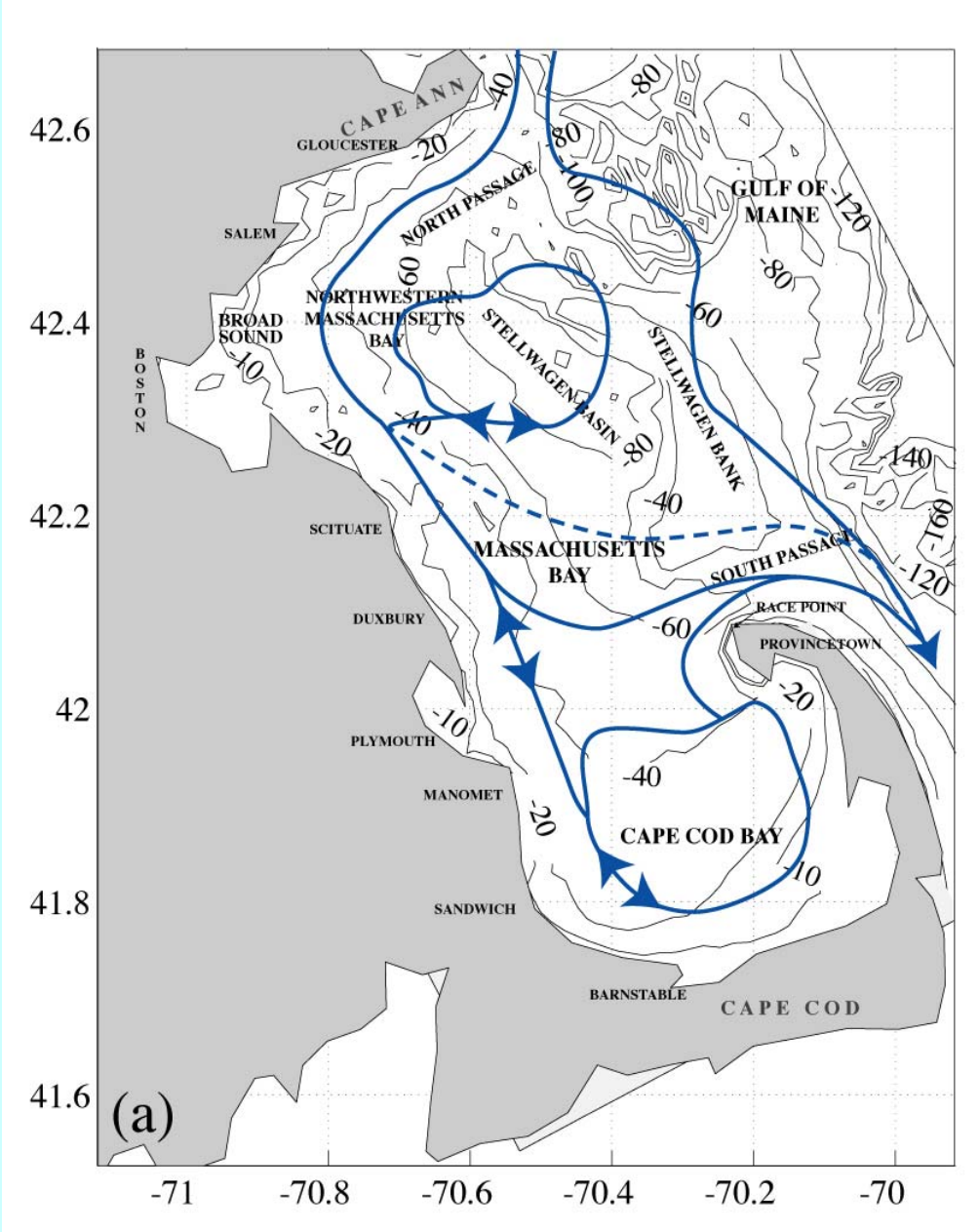
Acoustics: $\boldsymbol{x}_A = [\text{Pressure (p)}, \text{Phase } (\varphi)]$



$$\boldsymbol{P} = \varepsilon \left\{ (\hat{\boldsymbol{x}} - \boldsymbol{x}^t)\,(\hat{\boldsymbol{x}} - \boldsymbol{x}^t)^T \right\}$$

$$\boldsymbol{P} = \begin{pmatrix} P_{AA} & P_{AO} & P_{AB} \\ P_{OA} & P_{OO} & P_{OB} \\ P_{BA} & P_{BO} & P_{BB} \end{pmatrix}$$

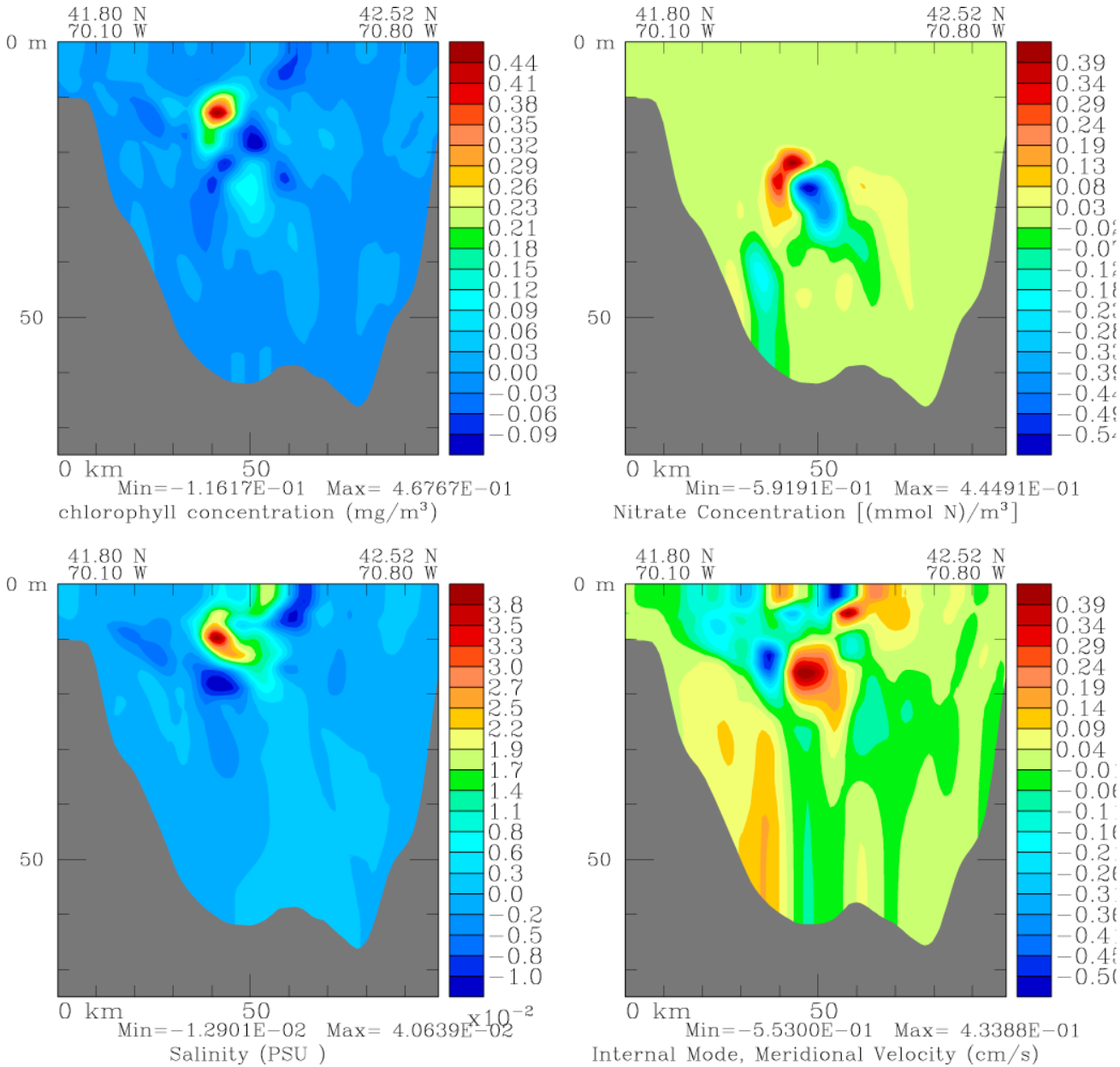
BIOGEOCHEMICAL-PHYSICAL SMOOTHING IN MASSACHUSETTS BAY



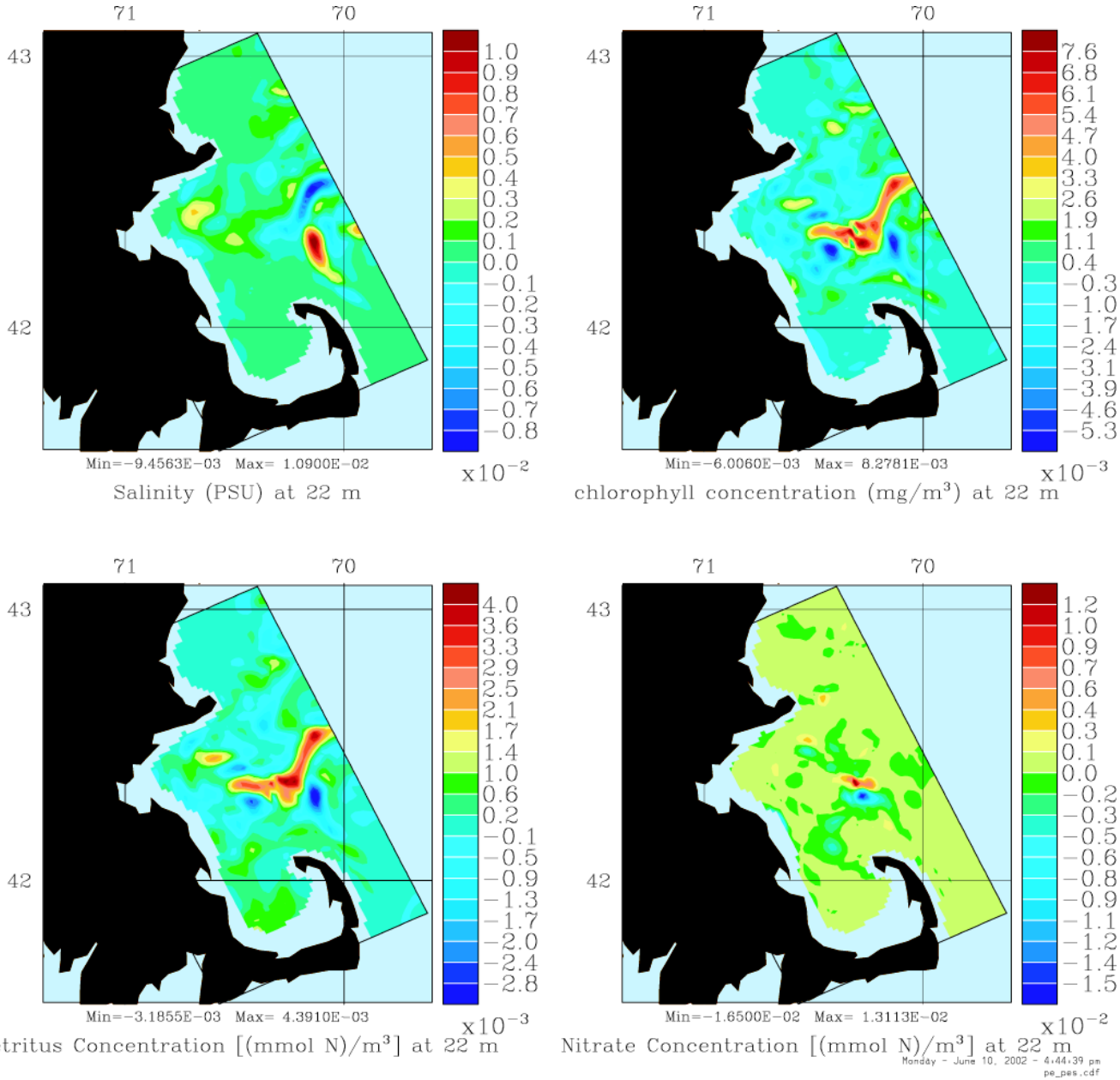
Cartoon of horizontal circulation patterns for stratified conditions in Massachusetts Bay, overlying topography in meters (thin lines).

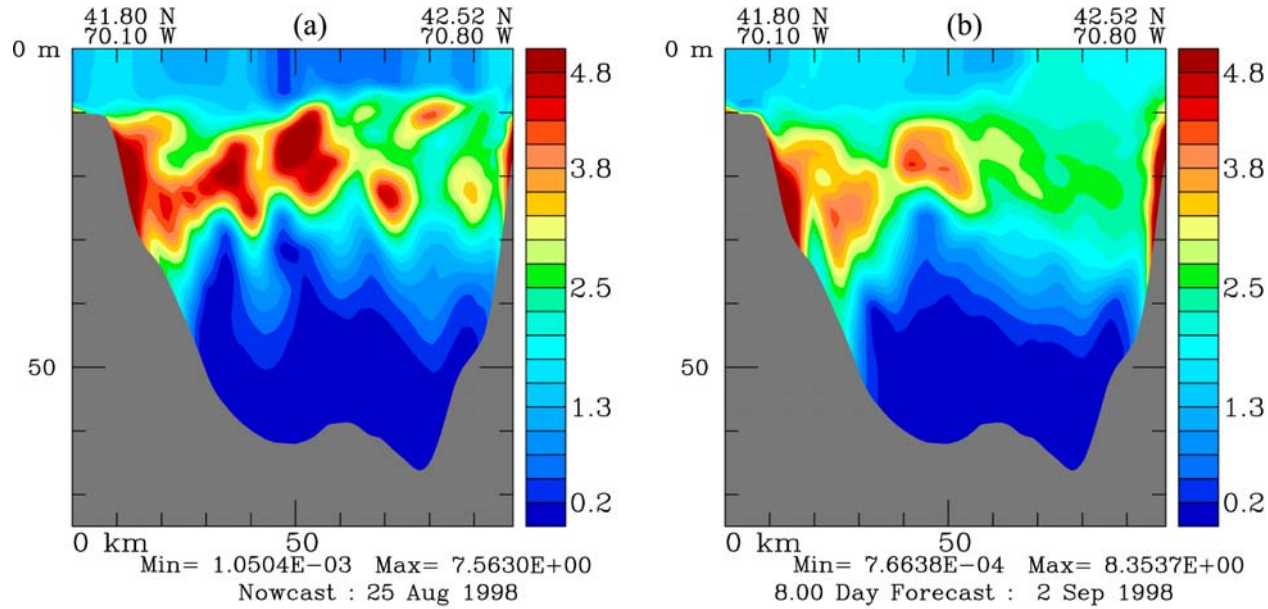
- **Patterns drawn correspond to main currents in the upper layers of the pycnocline where the buoyancy driven component of the horizontal flow is often the largest**
- **Patterns are not present at all times**
- **Most common patterns (solid), less common (dashed)**

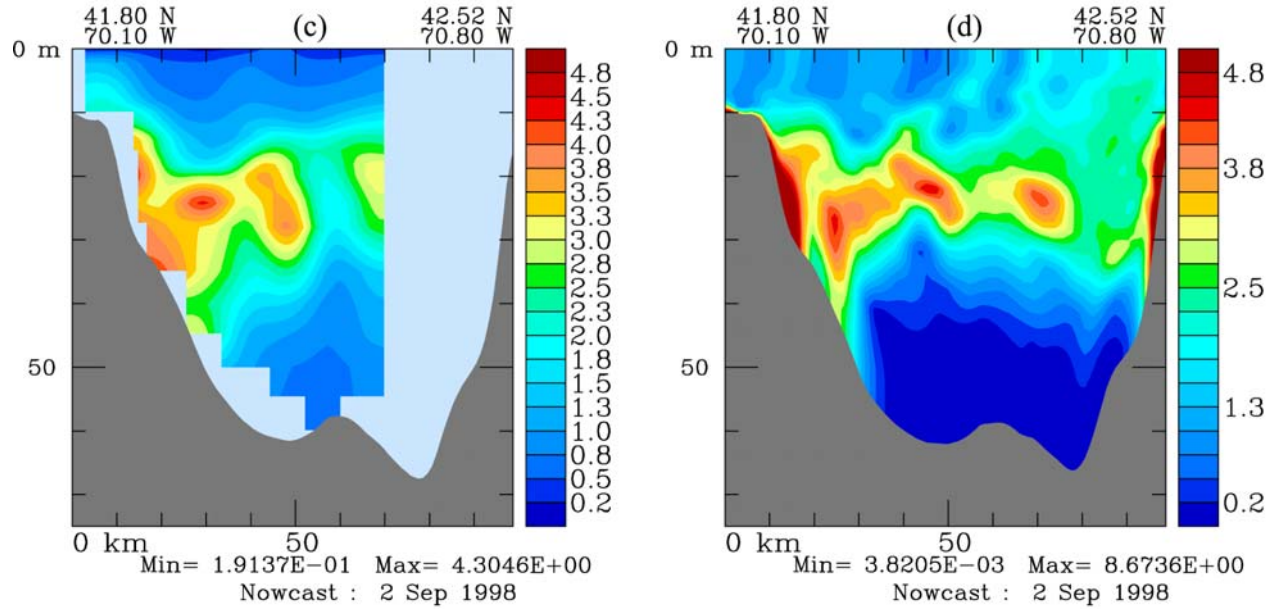
ESSE BIOGEOCHEMICAL-PHYSICAL ERROR COVARIANCE (FCST FOR SEP 2)



ESSE ERROR EIGENMODE 2 (FCST FOR SEP 2)







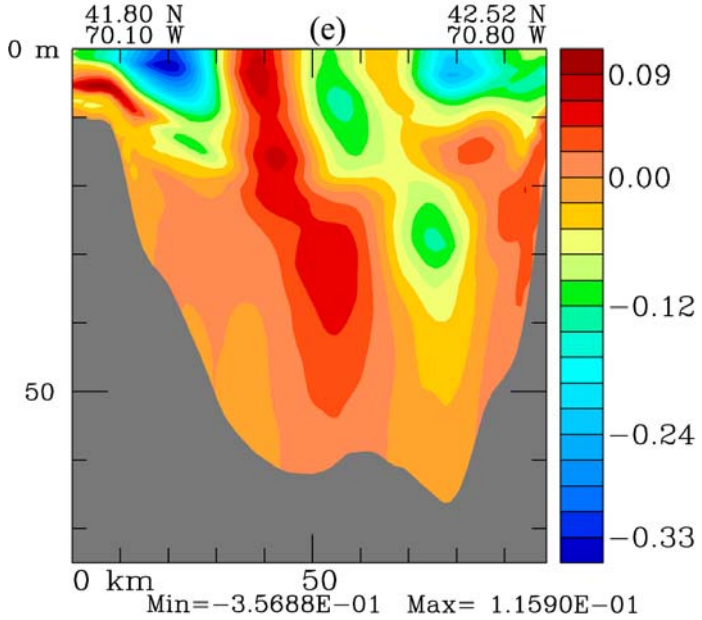
Cross-sections in Chl-a fields, from south to north along main axis of Massachusetts Bay, with:

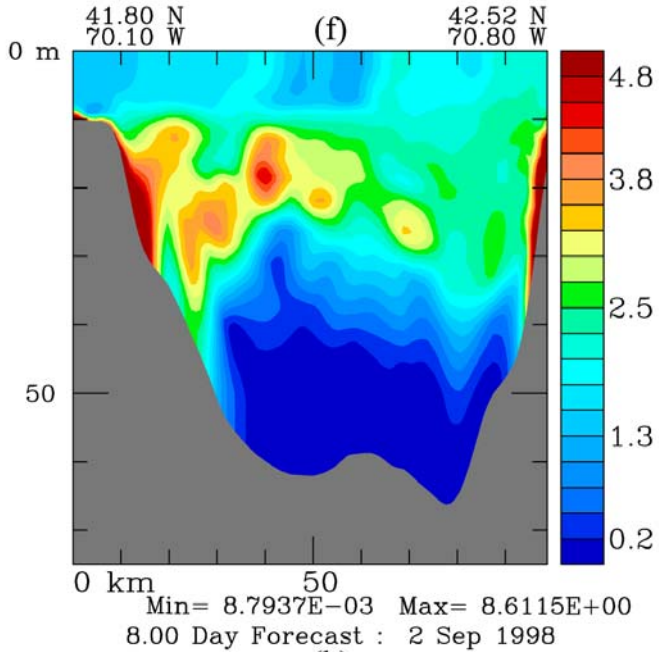
a) Nowcast on Aug. 25

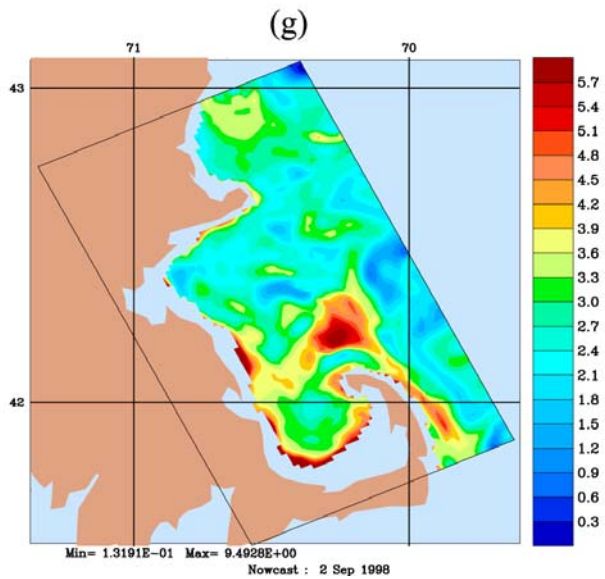
b) Forecast for Sep. 2

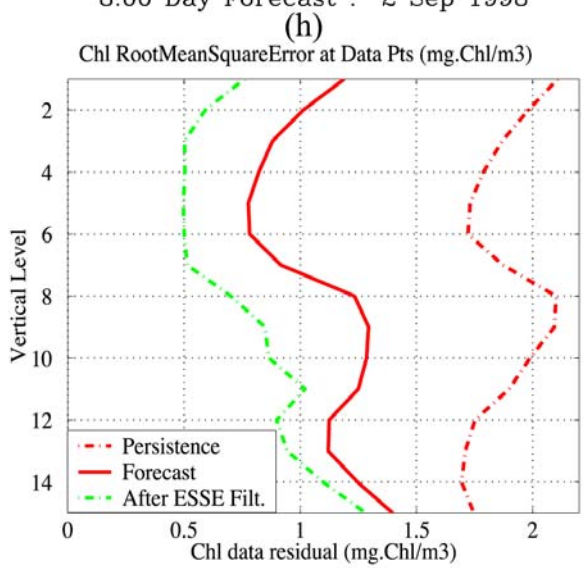
c) 2D objective analysis for Sep. 2 of Chl-a data collected on Sep. 2–3

d) ESSE filtering estimate on Sep. 2







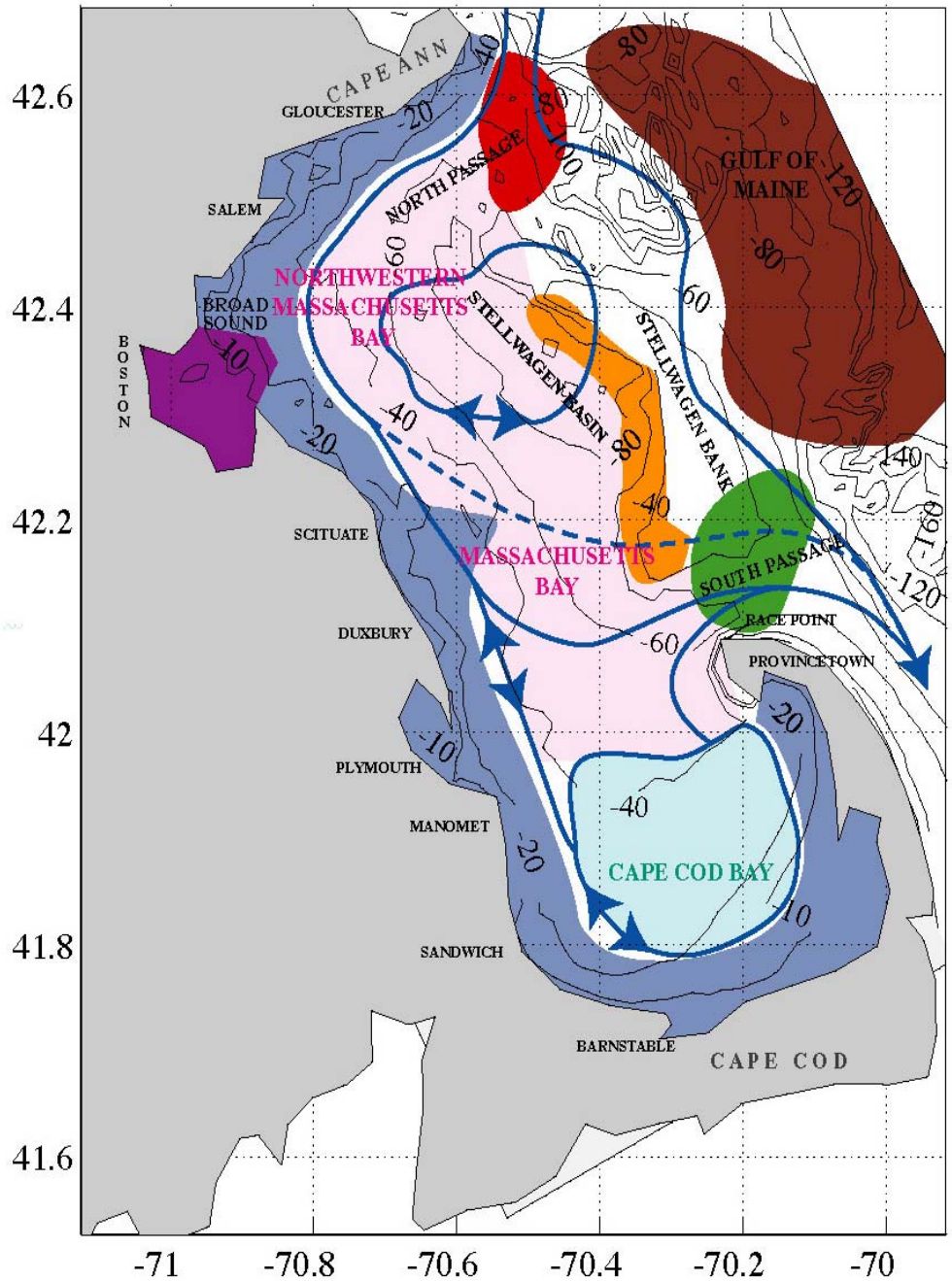


e) Difference between ESSE smoothing estimate on Aug. 25 and nowcast on Aug. 25

f) Forecast for Sep. 2, starting from ESSE smoothing estimate on Aug. 25

(g): as d), but for Chl-a at 20 m depth

(h): RMS differences between Chl-a data on Sep. 2 and the field estimates at these data-points as a function of depth (specifically, "RMS-error" for persistence, dynamical forecast and ESSE filtering estimate)



Coupled bio-physical sub-regions of Massachusetts Bay in late summer: Dominant dynamics for trophic enrichment and accumulation

Boston Harbor: Charles River, sediments, toxic material, NO_3–NH_4

Along Coast: upwelling/downwelling ⇒ bio ↑/↓

Open Bay: submesoscale/mesoscale eddies. Ageostrophic w ⇒ bio

Cape Cod Bay: Horizontal bio advection and submesoscales

West of Stellwagen Bank: GOM meanders, tides, topographic upwell/downwell

Offshore: GOM meanders

Race Point: Multiple bio advections, accumulation, and tides

Cape Ann: Physical instabilities at GOM inflow