Developing Efficient Navigation Schemes in Geophysical Flows

Geometric Mechanics and Turbulence Modeling Workshop, Nov. 8th-9th 2002

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Overview

- Our main objective is to aid in the design of an Autonomous Oceanographic Sampling Network by developing efficient navigation and control schemes for groups of Slocum Gliders

Slocum Glider

- Merge results from Dynamical Systems Theory with some results from Nonlinear Control Theory
- Adaptive navigation planning based on learning and utilizing the global geometry of the flow
- Direct Lyapunov Exponent (DLE) Contours used to obtain rough estimate of flow geometry, from which we extract efficient navigation routes and identify potential sampling hotspots

Computing DLE

Starting from the flow field

$$\dot{x} = f(x,t)$$

Let $x(x_0,t)$ denote a solution and define the strain tensor

$$\Sigma_{x_0} = \left[\frac{\partial x}{\partial x_0}\right]^T \left[\frac{\partial x}{\partial x_0}\right]$$

We define the (finite-time) Direct Lyapunov Exponent as

$$DLE_t(x_0,t_0) = \frac{1}{2(t-t_0)} \log(\sigma_t(x_0))$$

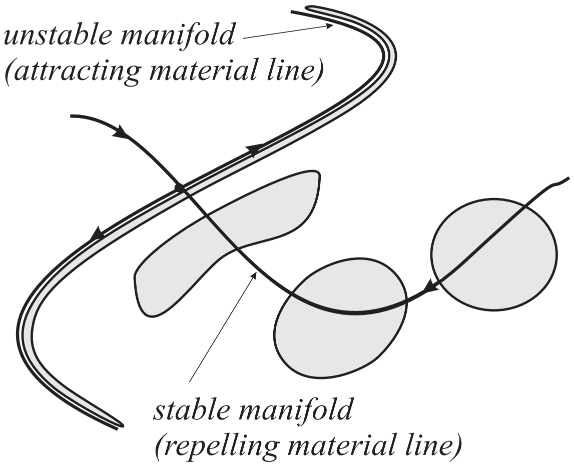
where $\sigma_t(x_0)$ is the maximal eigenvalue of Σ_{x_0}

How is DLE used?

- Computing DLE for grids of points produces time-dependent scalar fields
- Extrema in these scalar fields often correspond to stable and unstable manifolds [1],[2],[3]
- Computing DLE forward in time (i.e. $t - t_o > 0$) produces stable manifolds (aka ***repelling material lines***), while DLE computed backward in time reveals unstable manifolds (aka ***attracting material lines***)
- DLE contours have frequently been used to help understand mixing and transport [3],[4],[5]
- Advantages of DLE:
  - Simple to use--just look at contours
  - Reveals underlying flow structure that might otherwise be hard to detect from Eulerian velocity field
  - Robust--Attracting and Repelling Material Lines persist under perturbations to the Eulerian field. That is, *approximate* velocity data can still provide good results [2].

Terminology:

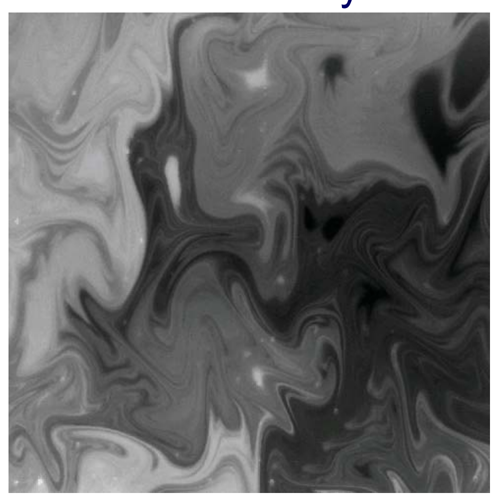
We refer to a stable manifold as a Repelling Material Line since it would tend to stretch a parcel of Lagrangian particles placed about it, whereas the unstable manifold is deemed an Attracting Material Line since the parcel would get attracted to it as shown to the right.



Application to mixing:

Shown to the right is the mixing of colored dyes. On the far right, is the same picture with the DLE contour superimposed. Ridges of high DLE are shown in red.

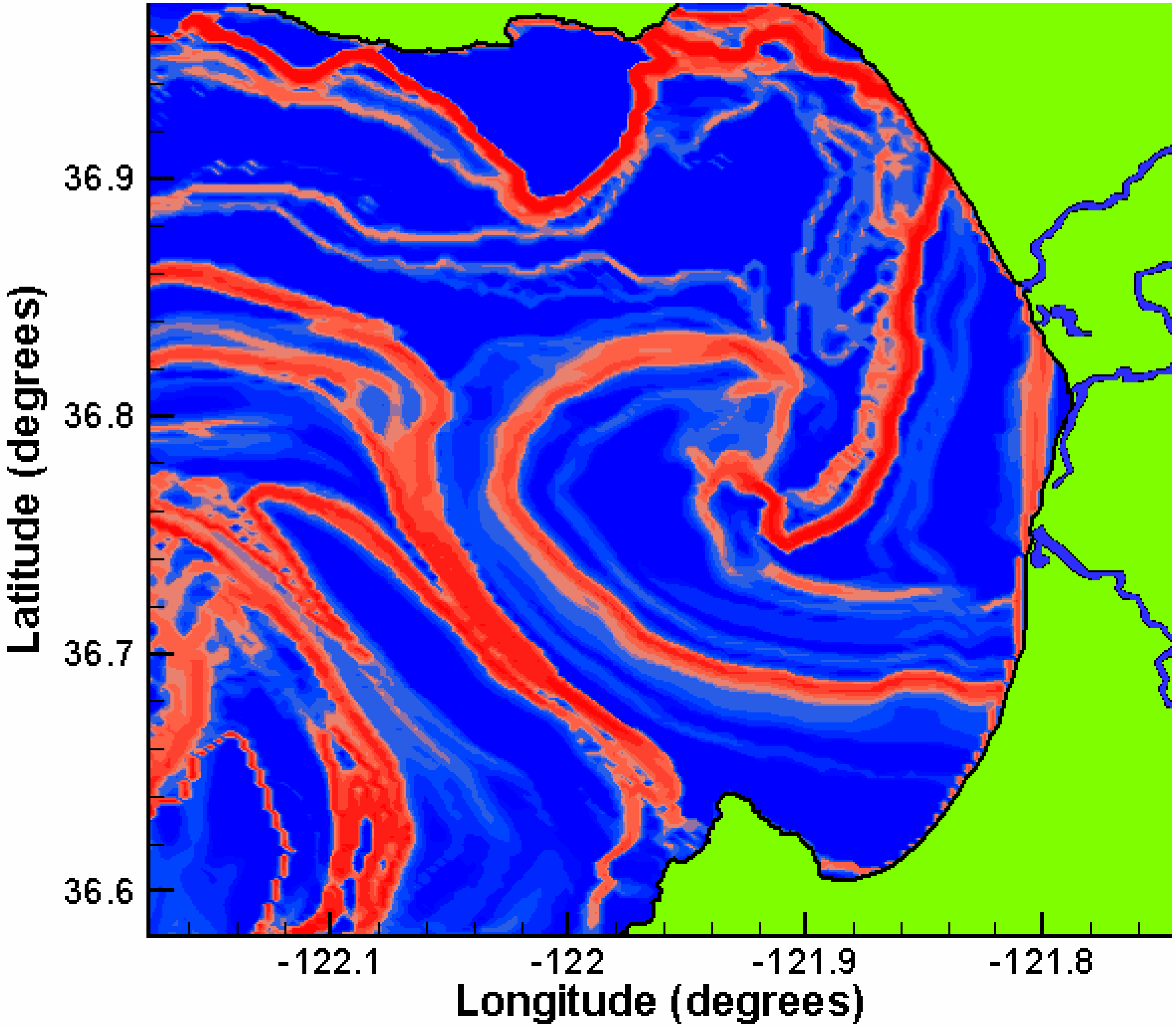
Diffusive dye



Attracting lines

Voth, Haller, & Gollub [PRL **18** (2002) 244501]

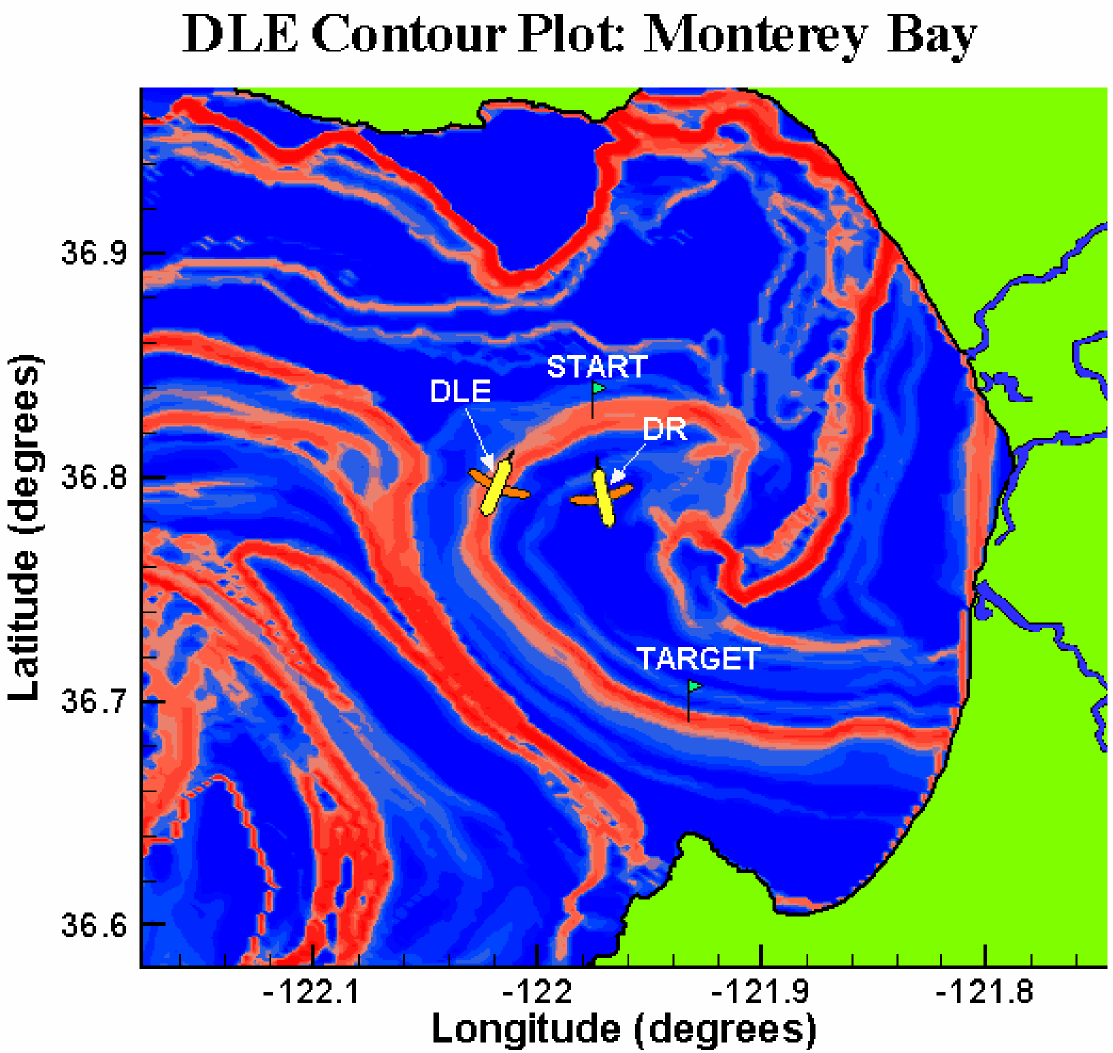
DLE Contour Plot: Monterey Bay



To the left is a snapshot of a typical DLE contour plot for Monterey Bay. The red pathways correspond to high DLE values (i.e. repelling material lines since DLE was computed forward in time) and the blue regions to low DLE values. The central c-shaped pathway is created from a counter-clockwise gyre emanating from the center of the bay.

Using DLE for Navigation

- Use Repelling Material Lines as navigational pathways to direct vehicles
  - Using stable/unstable manifolds for navigation has been applied to space mission designs, e.g. NASA's current Genesis Mission
- Procedure for Adaptive Sampling:
  - From an Ocean Prediction Model, estimate future flowfield and discern sampling hotspots
  - Use DLE contours computed from predicted flow to find efficient pathways to sampling hotspots



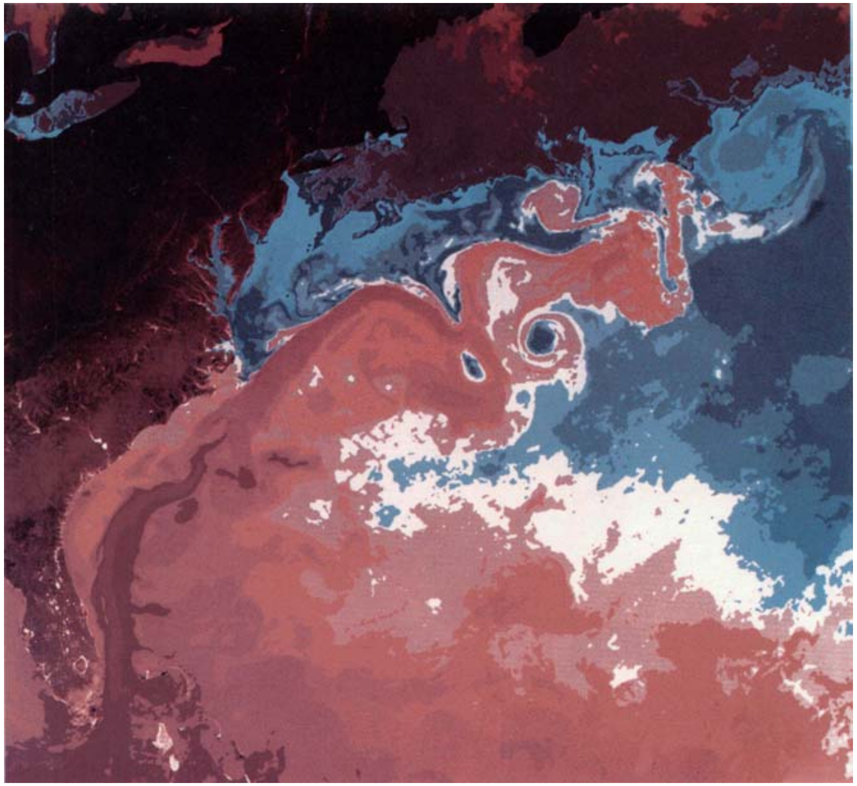
To the left is a cartoon of two gliders in Monterey Bay. The gliders begin from the start flag and are trying to reach the target flag. The glider labeled ‘DLE’ tries to reach the target via the high DLE ridge shown in red while the glider labeled ‘DR’ is just using a “dead-reckoning” approach where it continually tries to point itself in the direction of the target.

Using DLE to detect sampling hotspots

- There is evidence that supports DLE can be used to determine regions of sampling interest:

Attracting Material Lines = Sampling Hotspots

- Use Attracting Material Lines to track regions of interest (e.g. eddies, volumes of biomass)
- Use Repelling Material Lines to direct gliders to sampling hotspots (Attracting Material Lines)



Above is an infrared observation of the Atlantic Coast. Hot water (red) mixes with cold northern water (blue). The formation of cold rings can be explained by heteroclinic tangling of manifolds

Cooperative Control

- Use groups of vehicles to achieve cooperative sampling
- Coordinated sampling has several benefits
  - Decision making capability of group much more accurate than that of an individual
  - More effective gradient climbing
  - Redundancy
  - Etc, …
- Collective dynamics manipulated via Lyapunov-based methods [6]
  - Use Artificial Potentials with a Virtual Leader to control translation, rotation, and expansion of group

Future Directions

- Quantify benefits of using DLE for path planning over more naïve path planning algorithms
- Further investigate relations between sampling hotspots (as determined from biologists and oceanographers) and DLE contours
- Look for other applications of Dynamical Systems Threory to developing an *Autonomous* Sampling Network
- Extend dynamical system methods and ideas to 3-D

References

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3. Bowman, K.P., Large-scale isentropic mixing properties of the Antarctic polar vortex from analyzed winds,*J. Geophys. Res.*, *98*, 23,013-23,027, 1993b
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5. Haller, G., and A.C. Poje, Finite time transport in aperiodic flows, *Physica D*, *119*, 352-380, 1998
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