ELSEVIER

Deep-Sea Research II 51 (2004) 393–410

DEEP-SEA RESEARCH
PART II

www.elsevier.com/locate/dsr2

Frictional processes in straits

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Received 22 November 2002; received in revised form 9 July 2003; accepted 4 October 2003

Abstract

Friction and mixing can be dynamically significant in straits, as in many oceanographic situations. For weak time-dependent homogeneous flows, geostrophy, friction, and acceleration can all play roles in limiting the flow through a strait, and the ratio of the geostrophic elevation difference across the strait to the frictional difference along the strait may be of importance in more general situations. A specific question for quasi-steady exchange flows through straits concerns the influences of friction and entrainment on hydraulic control. Within the context of layered models, these effects tend to drive a flow towards hydraulic criticality in the downstream direction and to shift control points downstream. However, friction and entrainment also induce vertical gradients in velocity and density, rendering layered models dubious. In such situations, the conditions for hydraulic control have been uncertain; even for a homogeneous fluid there is an apparent contradiction between control conditions based on a similarity assumption for the velocity profile on the one hand and on the speed of long waves on the other. The resolution of this contradiction is discussed. It seems that if the frictional forces only involve local flow properties, then inviscid long waves are arrested at the control section (though this is shifted from its location for inviscid flow), but that if flow derivatives along the channel are involved, the condition changes. For a homogeneous flow with shear, internal friction, and bottom friction, the control section is shifted downstream partway to where it would be for a slab flow with bottom friction. While the parameterization of bottom friction actually seems to be reasonably well established, the nature and quantification of internal and lateral frictional processes are uncertain. In a rotating system, secondary cross-strait flows are expected to be driven by the vertical gradient of the vertical Reynolds stress and can provide estimates of the magnitude of bottom and internal friction.

1. Introduction

Resolving all the scales of motion that occur in the ocean is far beyond modern, or foreseeable, computing capability, even for comparatively simple situations. There thus seems to be a need to continue basic studies of the unresolved processes so that we can parameterize their effects in terms of those processes that are resolved in numerical models. On the other hand, there are situations where the behaviour of the ocean is rather insensitive to the effects of small-scale processes, and low-resolution models provide reliable results independently of uncertainty about the parameterization of unresolved processes. For flow through straits, the effects of unresolved processes, such as those giving rise to friction and mixing, may indeed be of secondary importance in some circumstances, though in others they may

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0967-0645/$ - see front matter
doi:10.1016/j.dsr2.2003.10.005

exert a first-order influence on the flow. One purpose of this paper will be to review studies that help us decide on the importance of the small-scale processes.

Even if a numerical model does capture all the essential physics of some aspect of ocean circulation, it is still important to develop an intuitive understanding of the output, thus, for example, thinking of the existence of the Gulf Stream in terms of vorticity arguments for western intensification rather than merely as a solution of the Navier–Stokes equations. This philosophy seems to be particularly appropriate for strait flows, where concepts such as hydraulic control and maximal exchange are powerful aids to understanding and, hence, prediction. It is thus particularly important to understand the effects of friction and mixing on these inviscid concepts.

In Section 2 of this paper I will briefly review homogeneous strait flows in a limit with negligible inertial effects, discussing the way in which both geostrophy and friction can limit the flow. Section 3 addresses the general influence of friction on steady layered flows through straits, starting with one layer and then extending the discussion to two layers. Brief mention will be made, in Section 4, of the internal effects of rotation. In Section 5 I add the effects of entrainment, focussing on a one-layer reduced gravity flow. While most of the discussion will be a review, recent results on the effect of entrainment will be shown to point to a new resolution of a long-standing puzzle concerning flow fluctuations in the Strait of Gibraltar. To this point the paper still will have retained the approximation of representing a flow by separate homogeneous layers. Many observations, however, clearly show that the effects of friction and entrainment lead to gradual rather than abrupt transitions between water masses. In Section 6 I will discuss the extent to which the concept of hydraulic control may be extended to allow for frictionally induced shear, again with some new results as well as review material.

The precise form for the parameterization of unresolved frictional and mixing processes will be discussed briefly in Section 7, along with an account of some observations of Reynolds stresses. It will be clear that there are significant gaps in our ability to provide reliable parameterizations. Some open questions will be summarised in Section 8.

2. The effect of friction

A simple situation to consider is for the time-dependent flow of homogeneous water through a strait of uniform depth h and width W. The sea-level slope along the strait is then related to a force balance between the pressure gradient, bottom friction, and acceleration. Only the time-dependent term of the acceleration matters if the advective terms are small. If the length of the strait is L, the total sea-level drop is just $(-\mathrm{i}\omega + \lambda/h)Lu/g$ if bottom friction is linear with coefficient λ and the variables are time-dependent and proportional to $\exp(-\mathrm{i}\omega t)$. If the flow is superposed on a much stronger oscillatory tidal current with peak speed u_{T}, then a quadratic bottom friction with drag coefficient C_{d} can indeed be linearized, with $\lambda = (4/\pi)C_{\mathrm{d}}u_{\mathrm{T}}$.

If the sea-level difference along a strait between two basins is $\mathrm{Re}\,[\Delta\zeta\exp(-\mathrm{i}\omega t)]$, then the total volume flux $\mathrm{Re}\,[Q\exp(-\mathrm{i}\omega t)]$ has

$$Q = \frac{WhL^{-1}g\Delta\zeta}{-\mathrm{i}\omega + \lambda/h}. \tag{1}$$

The relative importance of acceleration and friction is given by the ratio $\omega h/\lambda$ of the spindown time h/λ to the timescale ω^{-1}, but this is not specific to straits. What is relevant in the case of strait flows is that there is also a geostrophic sea-level difference across the strait of fWu/g. It seems implausible that this should be greater than $\Delta\zeta$.

Further analysis and discussion of this situation (e.g., Toulany and Garrett, 1984) led to (1) being replaced by

$$Q = \frac{WhL^{-1}g\Delta\zeta}{-\mathrm{i}\omega + \lambda/h + fW/L}, \tag{2}$$

where f is the Coriolis frequency. The flux is still given by (1) if $f = 0$, but for steady flow and no friction becomes $Q = gh\Delta\zeta/f$, as for the volume flux in a Kelvin wave of amplitude $\Delta\zeta$, independently of the width W. This limit was termed "geostrophic control".

Overall (2) thus seemed to present a nice transition between plausible limits. Moreover it was, shown to be correct for the diffraction of frictionless tidal waves through a gap between two semi-infinite basins, assuming linear theory (small amplitude), with $\Delta\zeta$ described as the elevation difference that would exist if the gap were closed (Toulany and Garrett, 1984), and with L replaced by an effective length related to the end correction in diffraction problems.

On the other hand, Pratt (1991) pointed out that this analysis no longer applies if advective terms are significant, as is increasingly likely at low frequency. In particular, a flow established from rest in response to a sea-level difference between basins will have a front that is eventually advected out of the strait. In the absence of friction, a steady flow through the strait can then develop with no sea-level difference at all between the basins along a streamline, though there will be a difference across the strait and across boundary currents in the basins. Care is clearly required in analysing how a flow is established, but, in any event, it does seem that one relevant factor in considering steady homogeneous flow through straits is the ratio of the sea-level difference across the strait to that along the strait. With friction linearized about the tides, as above, this ratio is $\pi f W h (4 C_d u_T L)^{-1}$. For small values of this parameter, a coastal Kelvin wave would presumably not escape through a strait it encountered on a side boundary, but largely jump across it and continue along the main coast. It is interesting that for typical values, say $f = 10^{-4}\ \mathrm{s}^{-1}$, $W/L = 0.1$, $h = 100$ m, $C_d = 0.002$, and $u_T = 0.5\ \mathrm{m\,s}^{-1}$, the ratio is O(1).

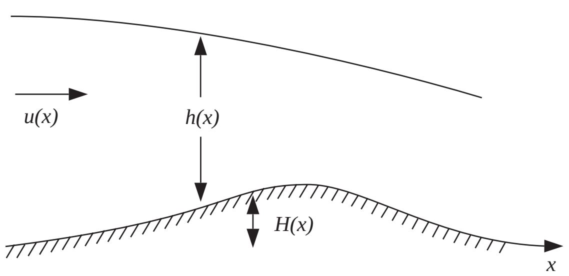

Fig. 1. Definition sketch for a single layer flow.

Apart from these general considerations for a uniform strait, most of the dynamics of interest in straits concerns the role of the advective terms within the strait itself, particular in response to varying topography and with the influence of stratification. We thus turn now to consideration of the influence of friction on hydraulics, with the term "hydraulics" denoting situations in which the flow speed is comparable with wave speeds.

3. The effect of friction on hydraulics

3.1. One layer

A simple problem to start with is that of steady non-rotating homogeneous shallow water flow down a channel of rectangular cross-section, with width $W(x)$ and bottom height $H(x)$ (Fig. 1). If the layer thickness is $h(x)$ and the current is $u(x)$, the governing equation for momentum is

$$u\frac{\mathrm{d}u}{\mathrm{d}x} + g\frac{\mathrm{d}}{\mathrm{d}x}(h+H) = -C_d\frac{u^2}{h} \qquad (3)$$

assuming quadratic bottom friction with drag coefficient C_d. This may be combined with the continuity equation

$$\frac{\mathrm{d}}{\mathrm{d}x}(uhW) = 0, \qquad (4)$$

which integrates to give $uhW = Q$, the constant volume flux.

A key parameter is the Froude number F, where $F^2 = u^2/(gh)$ and (3) may be rearranged to give

$$\frac{\mathrm{d}\beta}{\mathrm{d}x} = \frac{2}{3}\frac{\beta}{W}\frac{\mathrm{d}W}{\mathrm{d}x} - \left(\frac{gW^2}{Q^2}\right)^{1/3}\left(\frac{\mathrm{d}H}{\mathrm{d}x} + C_d F^2\right), \qquad (5)$$

where β is the usual hydraulic function $\frac{1}{2}F^{4/3} + F^{-2/3}$. This has a minimum of $\frac{3}{2}$ where $F = 1$ and $\mathrm{d}\beta/\mathrm{d}x = 0$, corresponding to a control point where the flow can switch from subcritical with $F < 1$ to supercritical with $F > 1$.

We see that the effect of friction is the same as that of decreasing width or increasing bottom height in pushing the flow towards criticality as one proceeds downstream. Also, a control point is shifted downstream of where it would be without friction. In particular, the control point occurs

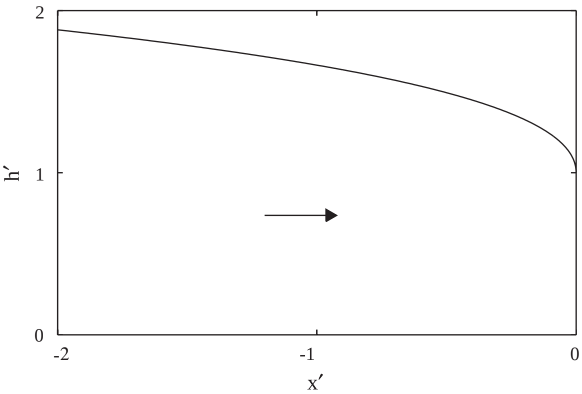

Fig. 2. The scaled free surface for a single-layer frictional flow, from left to right, through a uniform channel with exit control at $x' = 0$.

where the bottom slope $\mathrm{d}H/\mathrm{d}x = -C_\mathrm{d}$ if the width W is constant (Pratt, 1986) or at a location where $\mathrm{d}W/\mathrm{d}x = C_\mathrm{d}(W/h)$ if the bottom is flat, as is effectively the case for a reduced-gravity surface flow.

These results come from (3) using the simple identity $(gW^2/Q^2)^{1/3} = h^{-1}F^{-2/3}$. In fact, for this simple problem it is only necessary to follow one dependent variable since h and u are connected by volume flux conservation $uhW = Q$ and the Froude number may be written as $F = (h_0/h)^{3/2}$, where $h_0 = [Q^2/(gW_0^2)]^{1/3}$ is the layer depth at the control section where $W = W_0$.

One interesting problem concerns flow through a channel with a flat bottom and constant width. The control will be at the exit where the channel deepens or widens abruptly. Taking this to be at $x = 0$ and defining $x = (h_0/C_\mathrm{d})x'$ and $h' = h/h_0$, the equation for the surface slope is

$$\mathrm{d}h'/\mathrm{d}x' = -(h'^3 - 1)^{-1} \tag{6}$$

with the simple integral

$$x' = h' - \frac{h'^4}{4} - \frac{3}{4} \tag{7}$$

as illustrated in Fig. 2.

The surface is parabolic near the exit control at $x' = 0$ and acquires an infinite negative slope near there (thus violating the shallow water assumption, a problem that would disappear with more realistic topography), but the main point to make is that the horizontal scale for significant changes in layer depth is h_0/C_d. This result is easily established, of course, by scale analysis of the inertial and friction terms in (3). It provides a guide to the circumstances under which friction will be important, namely, if the channel is long compared with h_0/C_d. For a reduced gravity flow with $h_0 = 100$ m and an interfacial friction coefficient of, say, 5×10^{-4} (one quarter of a drag coefficient of 2×10^{-3} using the velocity difference to the centre of the interface between the active and stagnant layers), this distance is 200 km. Pratt (1986) tabulates an equivalent parameter for a number of straits and suggests that the Strait of Gibraltar is the only one on his list for which frictional effects are small.

3.2. Two layers

The one-layer situation provides a good guide to the effect of interfacial friction on a two-layer exchange flow through a channel of finite length but uniform cross-section, as discussed by Assaf and Hecht (1974). For a small density difference between the two layers, the condition for hydraulic control now becomes $F_1^2 + F_2^2 = 1$, where $F_i^2 = u_i^2/g'h_i$, for $i = 1, 2$ and u_i, h_i, the layer speed and thickness, and g' the reduced gravity based on the density jump between the two layers.

The effect of interfacial friction is to cause each layer to become thinner in the direction of its flow. Moreover, we note that equal and opposite volume fluxes imply that $h_1u_1 = h_2u_2$, so that if one layer is significantly thinner than the other, the Froude number of the thicker layer is likely to be very small and the active layer may be treated to a reasonable approximation as a reduced gravity flow, with applicability of the one-layer model. If one layer is thinner throughout the strait it may be controlled at its exit, but the control condition cannot be satisfied at the other end of the strait. Such a condition is termed "submaximal exchange", in that the flows are less than for "maximal exchange". This is the greatest that can be achieved for a given density difference between the two basins; it has exit controls at each end and subcritical flow in between. If friction is important, there will be a significant change in the thickness of each layer along the channel so that at each end the active layer will be much thinner than the other layer and in the neighbourhood of the exit controls the flow will thus again resemble that of a reduced gravity flow. Submaximal exchange can, of course, occur with the controlled layer not thinner than the other layer throughout the strait. It also needs to be remarked that if, as usually occurs, there are internal sills and narrows within a strait, control sections, if they exist, will tend to be near these rather than at the exits from the strait (e.g., Farmer and Armi, 1986; Bormans and Garrett, 1989).

One assumption that might well be questioned for both one- and two-layer flows is that of the uniformity of flow in the layers; we would expect bottom friction to induce a turbulent shear flow. I will return to this issue of shear later, but turn first to the role of rotation.

4. Rotation

The overall influence of rotation on hydraulic flows is reviewed in this volume by Larry Pratt. I will thus mention here only some simple considerations of the interplay between friction and rotation.

If the time taken for a single-layer flow through a channel is longer than the inertial period, a bottom Ekman layer will be established. The effect of friction on the interior flow is due to the Coriolis force acting on the secondary flow across the channel driven by the returning Ekman flux (Fig. 3), rather than by the vertical transfer of frictional stresses by turbulence. It is thus possible for the main flow, above the bottom Ekman layer, to remain laminar and slab-like while experiencing the effect of bottom friction. Stratification can confine the effect of friction near the bottom (Garrett and Petrie, 1981; Toulany et al., 1987).

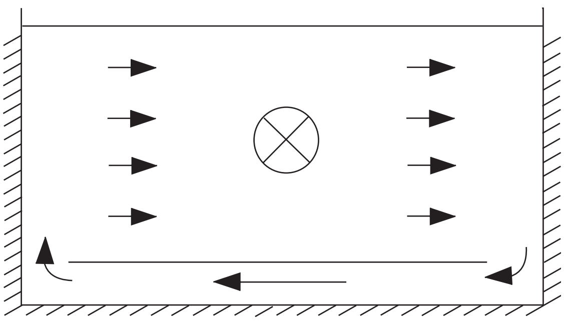

Fig. 3. Bottom friction in a rotating channel flow acts via a bottom Ekman layer and a returning cross-channel flow in the interior.

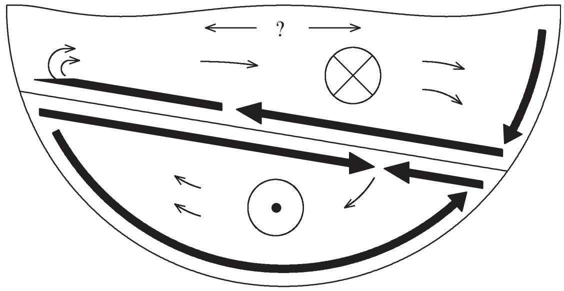

Fig. 4. The secondary cross-channel flows induced by bottom and interfacial friction in a two-layer exchange flow (Johnson and Ohlsen, 1994). The Ekman layer flows (thick arrows) are as for the Northern Hemisphere. The return flows in the layer interiors are illustrated by the thin arrows.

The effect of friction in a two-layer exchange flow has been examined in a laboratory experiment by Johnson and Ohlsen (1994). Their schematic of the flow (Fig. 4) shows the cross-channel interfacial tilt expected even for inviscid flows and also shows the presence of bottom and interfacial Ekman layers. The behaviour of the latter in the laboratory experiment was rather more complicated than expected, in that the interfacial Ekman layer just below the interface did not extend all the way to the side boundary, but seemed to meet an outflow from converging bottom boundary layers. Further investigation is warranted.

Most straits, of course, have sloping lateral boundaries rather than the flat bottoms and vertical sidewalls of the simplest models, and the stratification is continuous rather than layer-like. With sloping boundaries and in the presence of stratification and rotation it is possible that the bottom Ekman layer will be "arrested" as discussed by MacCready and Rhines (1991) and Trowbridge and Lentz (1991). In this situation, a near-bottom thermal wind brings the velocity just above the sloping bottom to zero and the flow becomes essentially frictionless. The time for arrest to occur is approximately $(C_d N/f)^{-1/2}(f/N)^2 s^{-2} f^{-1}$ for upwelling-favourable flow (and longer for downwelling-favourable flow), where N is the buoyancy frequency and s the bottom slope (e.g., Garrett et al., 1993). This time may be compared with the transit time for flow through a strait. Using $C_d = 2 \times 10^{-3}$, $N = 10^{-2}\ \mathrm{s}^{-1}$, $f = 10^{-4}\ \mathrm{s}^{-1}$, $s = 10^{-2}$ the arrest time is only 6 h which is likely to be less than a transit time, but is sensitive to the parameter values, particularly the bottom slope. Jungclaus and Vanicek (1999) found that the output of a numerical model of flow through the Vema Channel did indeed show the asymmetric boundary layer behaviour expected, with some features corresponding to observations.

It does seem that many existing dynamical ideas are important, though one weakness of the simple models is that they take a smooth bottom, ignoring the complications that might be introduced by the wide range of scales of bottom topography.

5. Entrainment

Internal friction in stratified shear flows is most likely to be accompanied by vertical mixing of density. Within a framework that treats the flow in terms of layers, this exchange of mass may be represented as entrainment. In particular, Csanady (1990) showed that, if the vertical mixing is parameterized by an eddy coefficient $K_v(z)$, then the upward entrainment speed across the isopycnal

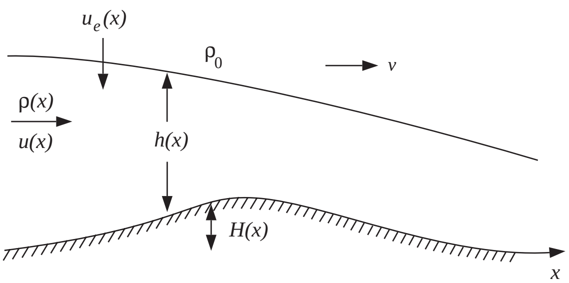

Fig. 5. A single-layer reduced gravity with an entrainment rate u_e from an inactive layer of density ρ_0 and speed v. (After Gerdes et al., 2002.)

located at the density inflection point, where $d^2\rho/dz^2 = 0$, is given by dK_v/dz evaluated at that depth.

As for friction alone, considerable insight into the effects of entrainment may be obtained from consideration of the flow of a single active layer. Gerdes et al. (2002) have discussed a reduced gravity single-layer flow beneath (Fig. 5) or above a deep inactive layer with constant speed v. For simplicity, I will take $v = 0$ in this review.

The volume conservation equation giving changes in $Q = uhW$ is now $dQ/dx = u_e W$ and mass conservation requires $d(\rho Q)/dx = \rho_0 u_e W$. Together these imply that $g'Q =$ constant, where $g' = g(\rho - \rho_0)/\rho_0$ is the reduced gravity. Consideration of momentum conservation in a control volume leads to relationships for the evolution of flow properties. The governing equations are simplified if one makes the Boussinesq approximation of ignoring the difference between ρ and ρ_0 unless it is multiplied by g.

Unlike the non-entraining case in which it was sufficient to have an evolution equation for just one dependent variable, we now need two. Choosing these to be the Froude number F, where $F^2 = u^2/g'h$, and the layer depth h, Gerdes et al. (2002) show that

$$\frac{dF^2}{dx} = \frac{-3F^2}{h(1-F^2)}\left[\frac{(2+F^2)}{3}\frac{h}{W}\frac{dW}{dx} - \frac{dH}{dx} - F^2 C_d - \left(\frac{1}{2}+F^2\right)\frac{u_e}{u}\right]. \tag{8}$$

It is clear that entrainment acts in qualitatively the same way as friction in driving the flow towards criticality. The equation for layer depth h is

$$\frac{dh}{dx} = \frac{1}{(1-F^2)}\left[F^2\frac{h}{W}\frac{dW}{dx} - \frac{dH}{dx} - F^2 C_d + \left(\frac{1}{2}-2F^2\right)\frac{u_e}{u}\right]. \tag{9}$$

This is interesting because it shows that entrainment may *thicken* a subcritical flow with $F < \frac{1}{2}$ even though the Froude number is increasing and friction thins the layer.

Reduced gravity flow over a sill, through a channel of constant width and with a stagnant upper layer, will now have a control shifted downstream to a location where $dH/dx = -C_d - \frac{3}{2}u_e/u$. The second term may be comparable with the first if, as Gerdes et al. (2002) discuss, $u_e/u = 0.002F^2$ from the entrainment law proposed by Christodoulou (1986). Similarly, the entrainment and friction terms may be of comparable importance in (8) and (9) at other locations. If u_e is increased by the action of things like breaking solibores (e.g., Wesson and Gregg, 1994), entrainment could dominate.

One final interesting consequence of adding entrainment to a reduced gravity flow occurs for the sea surface elevation ζ in a reduced gravity flow at the surface (such as occurs, for example, with the Atlantic water inflow into the Mediterranean Sea at the eastern end of the Strait of Gibraltar where the lower layer is deep, slow moving, and thus effectively stagnant). It is simple to derive

$$\frac{d\zeta}{dx} = \frac{g'}{g}\left(\frac{dh}{dx} - \frac{u_e}{u}\right) \quad (10)$$

$$= \frac{g'}{g(1-F^2)}\left[F^2\frac{h}{W}\frac{dW}{dx} - F^2C_d - \left(\frac{1}{2}+F^2\right)\frac{u_e}{u}\right], \quad (11)$$

where there is now no term in H, here or in (8) and (9), since the lower layer is passive. Entrainment may lead to a *decrease* in ζ for subcritical flow, even when widening would cause an increase. Moreover, the different coefficients in the u_e/u term in (9) and (11) show that entrainment can cause the sea surface to slope down at the same time as it is acting to increase the thickness of the upper layer. This seems contrary to the expectation of opposing surface and interface slopes if the lower layer is stagnant; it is a consequence of the downstream increase in density of the upper layer caused by entrainment.

5.1. Fluctuations in the Strait of Gibraltar

The result given by (11) may, in fact, help to resolve a long-standing mystery in connection with the nature of the exchange flow through the Strait of Gibraltar (Fig. 6).

Many pieces of evidence pointed to a seasonal switch between maximal exchange, with both Atlantic inflow and Mediterranean outflow controlled, and submaximal exchange with just the outflow controlled, as discussed by Garrett et al. (1990) (though see Ross et al., 2000, for a discussion of major interannual changes). However, the ratio of the fluctuations of sea-level differences across and along the eastern end of the strait seemed to agree year round with the expectations for maximal flow (Bormans and Garrett, 1989). This is shown in Fig. 7.

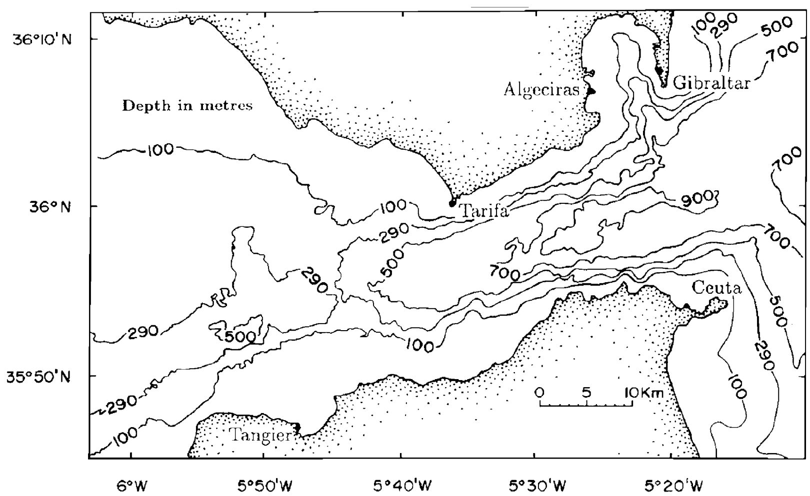

Fig. 6. The Strait of Gibraltar.

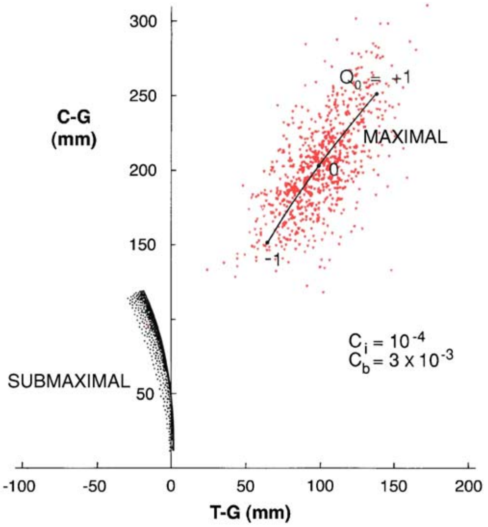

Fig. 7. Fluctuations in sea-level drops along the strait from Tarifa to Gibraltar (T-G) and across the strait from Ceuta to Gibraltar (C-G). The theoretical curves (Bormans and Garrett, 1989) for maximal and submaximal exchange are for changes induced by varying the barotropic flow with a range of ± 1 Sverdrup and, for the submaximal case, also varying the interface depth at the eastern end of the strait. (The submaximal predictions thus occupy a region.) Values of the interfacial drag coefficient C_i and the bottom drag coefficient C_b used in the model are shown. The data points, plotted here to be centred on $Q_0 = 0$ for maximal exchange, are for low-passed data, with a cut-off period between 39 and 28 h, from September 1981 to September 1982. The Tarifa and Gibraltar data have been adjusted for atmospheric pressure in order to give the subsurface pressure gradient; this was found to be unnecessary for C-G. Both T-G and C-G have also been slightly corrected to remove an apparent influence from direct forcing by local winds, a factor omitted from the models (see Garrett et al., 1989 for details).

The model curves show the expected fluctuations in the sea-level differences associated with the effect of the barotropic fluctuations (assumed to be mainly driven by atmospheric pressure changes over the Mediterranean) on a two-layer exchange flow in which each layer is treated as a uniform slab subject to bottom friction (acting on the sloping sides of the upper layer as well as the bottom beneath the lower layer) with coefficient C_b here and interfacial friction with coefficient C_i. The value of the latter was chosen to be compatible with the dissipation data of Wesson and Gregg (1994). The effects of entrainment were not included. The curve for maximal exchange ranges over barotropic fluctuations Q_0 between ± 1 Sverdrup (10^6 m^3 s^{-1}), with the positive sign being inflow from the Atlantic to the Mediterranean.

The slope of the major axis of the cloud of data points is positive and apparently compatible with the theoretical expectation for maximal exchange, so, in the absence of absolute levelling information, the data points have been plotted with the origin of the Tarifa to Gibraltar (T-G) and Ceuta to Gibraltar (C-G) fluctuations at the point on the maximal exchange curve for $Q_0 = 0$, i.e., the centre of the cloud of data points has been placed on the maximal exchange curve at the point where the flow fluctuation Q_0 is zero. The agreement appears reasonable.

On the other hand, as discussed by Garrett et al. (1989), the rms value of Q_0 required to produce the observed variances of C-G and T-G was about 60% greater than expected or observed. Moreover, the sea-level difference from Atlantic to Mediterranean strongly suggested that the exchange was submaximal for the summer of 1982, this being the latter part of the period considered in Fig. 7. Given the lack of absolute levelling, we could in fact slide the data points in Fig. 7 down and to the left to straddle the theoretical curves for maximal and submaximal exchange, but these curves still seem too far apart for this to be very plausible, and the slope for the submaximal solutions is wrong.

This slope was, in fact, mainly because of an expectation of increasing sea-level along the eastern end of the strait for the subcritical flow there in conditions of submaximal exchange. We see from (11) that this slope is affected by friction, where C_d in (11) lumps together the effect of both bottom and interfacial friction in the model of Bormans and Garrett (1989). Thus, as discussed by Garrett et al. (1990), increasing the interfacial drag coefficient in the model could swing the submaximal curve in Fig. 7 around to have a positive slope and also move it and the maximal curve closer together. An increase in internal friction would, however, seem to be incompatible with the dissipation data which, as mentioned above, led to the value chosen. Also, a much larger friction coefficient would move the control section downstream from near Tarifa Narrows to the section at Gibraltar, contrary to data indicating that the flow can be significantly supercritical by this stage. We seek another reason for the negative sea-level slope at the eastern end of the strait even in conditions of subcritical exchange and so decreasing Froude number. We now see from (11) that entrainment can have an effect on the sea-level slope, and, at small Froude numbers, could easily be more effective than the drag. We need an estimate of u_e/u.

This may be obtained from the study by Bray et al. (1995), who clearly document the way in which the Atlantic inflow to the Mediterranean entrains significant amounts of Mediterranean water that was otherwise heading out towards the Atlantic. Bray et al. (1995) analysed the situation by introducing an interface layer that has an increasing transport as it flows into the Mediterranean, having acquired it from the basic layers above and below it. Their summary of the along-strait and inter-layer transports is shown here in Fig. 8.

The data suggest an upward entrainment from the lower layer in the eastern part of the strait with $u_e/u \simeq 10^{-3}$, taking u as the average inflow in the surface and intermediate layers. This entrainment rate is perhaps larger than the Christodoulou (1986) formula $u_e/u = 0.002F^2$ if one bases an estimate of F^2 on a layer depth to the middle of the interface layer, giving $F^2 \simeq 0.1$. Enhanced entrainment is, in fact, likely to be associated with the mixing caused by the passage of internal solitary waves, as shown by Wesson and Gregg (1994).

For these values, and treating the inflow as a reduced gravity flow with $(h/W)\mathrm{d}W/\mathrm{d}x \simeq 0.002$, we see that, in comparison with the term in $\mathrm{d}W/\mathrm{d}x$, the entrainment term in (11) can be important for the along-strait sea-level gradient in conditions of submaximal exchange, and exceed the influence of the drag coefficient, while not having a significant influence in (8) on the evolution of the Froude number itself. Pursuing this further is probably not warranted given the limitations of the models, but we provisionally conclude that what had appeared to be a puzzle may not be so strange: entrainment may have a substantial influence on the submaximal solution shown in Fig. 7, rotating it around to have a positive slope and moving the whole curve up closer to the maximal solution. The cloud of data points, which we have argued is too large to

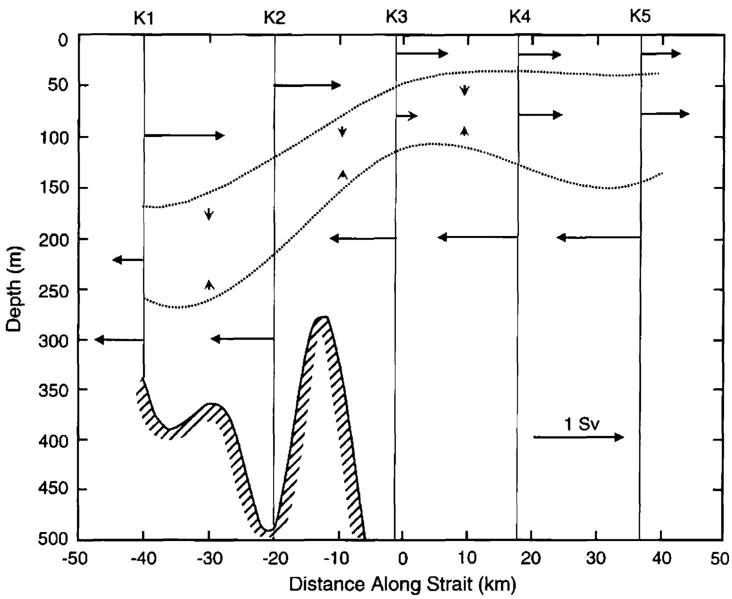

Fig. 8. Transports at the eastern end of the Strait of Gibraltar. The dotted lines show the interfaces between the interface layer and the basic layers of inflowing Atlantic water and outflowing Mediterranean water. (From Bray et al., 1995.)

correspond just to fluctuations on a basically maximal exchange state, can then encompass both maximal and submaximal conditions.

One major weakness of this discussion is that, although it allows for entrainment, it still treats the flows as being slab-like in both density and velocity, with no significant density gradient or velocity shear in the layers. We turn to this next, with particular attention to the role of shear in the velocity profile.

6. Beyond layers

One does not have to look far in data or in the output of numerical models to realise that the assumption of discrete, slab-like, layers is dubious. Vertical profiles of density and horizontal velocity often, maybe usually, show significant gradients. These gradients may, of course, be a consequence of friction and entrainment.

We have already discussed the observed situation for the Atlantic inflow through the Strait of Gibraltar, where there is clearly an interface layer that really makes up part of the inflow. Another example is in Bab el Mandab at the entrance to the Red Sea. As discussed by Pratt et al. (2000), the density and velocity profiles there are highly sheared.

Interface layers were also generated in the numerical simulation by Winters and Seim (2000) of the exchange through a constriction of two different water masses. Hogg et al. (2001a) examined the problem further, finding that as the vertical mixing of mass and momentum is increased the exchange flow evolves from one resembling two-layer hydraulic exchange to one in which turbulence plays a dominant role. For a contraction of length L and height H, with a vertical eddy viscosity v, they find that the key dimensionless parameter is $Gr_T A^2$, where Gr_T is the turbulent Grashof number, given by $g'H^3/v^2$, with g' based on the density difference between the two reservoirs, and $A = H/L$. The hydraulic limit occurs for large values of this parameter. We note that, taking $U = (g'H)^{1/2}$ as a velocity scale, $(Gr_T A^2)^{-1/2}$ may be written as the ratio of viscous forces of order vU/H^2 to an inertial term of order U^2/L, in much the same way that this comparison is represented by $C_d L/H$ for one-layer flow.

The overall physics found in observations and numerical models is not surprising, with the main challenge appearing to be the determination of correct parameterizations of the vertical exchange of mass and momentum. I will return to this later, but first remark that another challenge is to understand the criteria for hydraulic control in models with continuous profiles of density and velocity. In the inviscid case with no mixing, Killworth (1992) found that long waves are arrested at a control section. The situation with internal friction and mixing is discussed by Hogg et al. (2001b) who examined the propagation of waves through a stratified shear flow established by "lock exchange" through a constricted channel. They found an extension of the usual concept of control, in that the various waves that exist do not seem to be able to carry information about the interface depth upstream through the constriction, but the situation is complicated.

I will next address the much simpler situation in which there is no density stratification and one seeks merely to extend the simple hydraulic theory, for a homogeneous fluid with a free surface, to allow for shear in the current. This will raise an apparent paradox that has been discussed recently by Garrett and Gerdes (2003).

6.1. Control conditions in a sheared flow

We assume that the horizontal scale is much greater than the water depth so that the hydrostatic assumption still applies, but we have to include in the momentum equation the advective term involving the vertical shear. The momentum equation is then

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + g\frac{\mathrm{d}}{\mathrm{d}x}(h+H) = \frac{\partial \tau}{\partial z}, \tag{12}$$

where τ represents internal frictional forces. The continuity equation is simply $\partial u/\partial x + \partial w/\partial z = 0$.

One situation that has been considered in the literature (Chow, 1959; Henderson, 1966) is with the assumption that the horizontal velocity component u maintains a similar shape, with horizontal variations only of the depth-averaged speed

$\bar{u}(x)$. Thus

$$u(x,z) = \bar{u}(x)P(\zeta), \quad \text{where } \zeta = \frac{z-H}{h} \tag{13}$$

and $P(\zeta)$, with $\int_0^1 P\,\mathrm{d}\zeta = 1$, is just a function describing the velocity profile.

Under these conditions the vertical integral of (12) and the continuity equation is just

$$M_2\bar{u}\frac{\mathrm{d}\bar{u}}{\mathrm{d}x} + g\frac{\mathrm{d}}{\mathrm{d}x}(h+H) = -\frac{\tau_\mathrm{b}}{h},$$
$$\frac{\mathrm{d}}{\mathrm{d}x}(h\bar{u}) = 0, \tag{14}$$

where we have assumed a constant channel width for simplicity, $\bar{u}(x)$ is the depth-averaged speed, $M_2 = \int_0^1 P^2\,\mathrm{d}\zeta$ and τ_b is the bottom drag. These are very similar to the equations governing a slab-like flow with bottom friction, but the presence of the scale factor M_2 means that control is achieved when the Froude number $\bar{u}/(gh)^{1/2}$, based on the depth-averaged current, is $M_2^{-1/2}$. This is less than 1. If the bottom friction can be represented in terms of the local depth-averaged current as $C_\mathrm{d}\bar{u}^2$, the control is shifted downstream of a ridge crest to a location where $\mathrm{d}H/\mathrm{d}x = -M_2^{-1}C_\mathrm{d}$.

The control condition may be compared with the expectation based on the speed of inviscid long waves. To derive this we start with the linearized momentum equation for a uniform channel of constant mean depth h. This is

$$\frac{\partial u'}{\partial t} + u\frac{\partial u'}{\partial x} + w'\frac{\mathrm{d}u}{\mathrm{d}z} + g\frac{\partial \zeta}{\partial x} = 0, \tag{15}$$

where a perturbation velocity (u', w') is superimposed on the basic sheared flow $u(z)$ and the surface elevation perturbation is ζ.

Seeking a wave-like solution with the perturbation variables proportional to $\exp[\mathrm{i}k(x-ct)]$ and using the continuity equation $\partial u'/\partial x + \partial w'/\partial z = 0$ leads to

$$(c-u)\frac{\partial w'}{\partial z} + w'\frac{\mathrm{d}u}{\mathrm{d}z} + g\mathrm{i}k\zeta = 0. \tag{16}$$

After dividing (16) by $(c-u)^2$, the left-hand side may be written as a differential. Integrating the resulting equation vertically and using the free surface kinematic boundary condition $w' = \partial\zeta/\partial t + u\,\partial\zeta/\partial x$ at $z = h$ we obtain

$$g\int_0^h (c-u)^{-2}\,\mathrm{d}z = 1. \tag{17}$$

This result is well established (e.g., Freeman and Johnson, 1970) but has been derived here to illustrate the simplicity of the derivation. It clearly gives $c^2 = gh$ if $u = 0$, and $(c-\bar{u})^2 = gh$ if u is independent of depth. For a small departure from depth-uniform flow, and taking $\bar{u} = 0$ for convenience, we take $u = (gh)^{1/2}\varepsilon(z)$ where $\int_0^h \varepsilon\,\mathrm{d}z = 0$. Expansion of the integrand in (17) then gives

$$c^2 = gh\left(1 + 3h^{-1}\int_0^h \varepsilon^2\,\mathrm{d}z\right) \tag{18}$$

as effectively given by Baines (1995, p. 54) where it is derived from the Taylor–Goldstein equation in the limit of zero stratification. Thus the magnitude of the speed of long waves is greater than $(gh)^{1/2}$ and is clearly sufficient to carry information upstream through an alleged control point at which the mean flow is, as shown earlier, less than $(gh)^{1/2}$.

6.2. Resolving the paradox

Garrett and Gerdes (2003) resolved the apparent paradox raised above by pointing out that the presence of internal friction in (12), along with the assumption of a constant shape of the velocity profile, changes the speed of long waves from that implied by the inviscid analysis. They show that the form of internal friction required to preserve the shape of the velocity profile is

$$\frac{\partial \tau}{\partial z} = (P^2 - M_2)\bar{u}\frac{\mathrm{d}\bar{u}}{\mathrm{d}x} - \frac{\tau_\mathrm{b}}{h}. \tag{19}$$

Using this on the right-hand side of (12) changes the long-wave speed (for $\tau_\mathrm{b} = 0$) to the value $M_2^{-1/2}(gh)^{1/2}$, as derived above. The reason for this change in long-wave speed from that for inviscid waves is that the right-hand side of (15) acquires an extra "frictional" term $(P^2 - M_2)\bar{u}\,\mathrm{d}\bar{u}'/\mathrm{d}x$, which depends on the flow derivative. It thus alternates in sign, providing no net damping for the wave but changing its propagation speed. These conclusions would need to be adjusted if τ_b depended on flow derivatives rather than just the local flow properties. In that case it, too, could be moved to the left-hand side of the momentum equation, with consequent changes in the long-wave speed and control condition.

6.3. Control of inviscid shear flow

So far we have established that a shear flow with a velocity profile of fixed shape is controlled when the depth-averaged flow is less than $(gh)^{1/2}$, whereas inviscid long waves on a shear flow have a speed, relative to the depth-averaged flow, greater than $(gh)^{1/2}$. Garrett and Gerdes (2003) have shown that an invisicid shear flow is actually controlled at this higher flow speed. The derivation is interesting and will provide a lead into a more general situation.

We start by noting that (12) with no friction, combined with the continuity equation $\partial u/\partial x + \partial w/\partial z = 0$ so that we may write $u = \partial\psi/\partial z$ and $w = -\partial\psi/\partial x$, may be integrated to

$$\tfrac{1}{2}u^2 + gh = g[a(\psi) - H]. \qquad (20)$$

This is just Bernoulli's equation for a vortical flow, with a a function of the streamfunction ψ rather than constant as for the unsheared flow.

The problem may then be cast in terms of a functional connection between h and H by writing the continuity equation as

$$h = \int_H^{H+h} \mathrm{d}z = \int_0^Q \frac{\mathrm{d}\psi}{u}, \qquad (21)$$

since $u = \partial\psi/\partial z$ and Q is the volume flux. This assumes a single-valued connection between ψ and z, equivalent to assuming a unidirectional flow. Combining (20) and (21) leads to

$$J(h;H) = \int_0^Q \frac{\mathrm{d}\psi}{(2g)^{1/2}[a(\psi) - (H+h)]^{1/2}} - h = 0. \qquad (22)$$

This is of the form $J(h;H) =$ constant required for the applicability of Gill's (1977) arguments based on the existence of a functional relationship between a single flow variable (h here) and external factors (just H here).

Garrett and Gerdes (2003) investigated the form of the functional $J(h;H)$ for sheared as well as unsheared flows. For example, Fig. 9 shows a non-dimensionalized $J(h;H)$ for $a(\psi) = a_{\min} + A\psi$ with $A = 1.5$. This form of a corresponds to a linear shear with $\partial u/\partial z = A$. Control occurs when the two possible solutions of (20) coalesce, as happens when a minimum of $J(h;H)$ as a function of h occurs on the h axis. For the example of Fig. 9, this is with $b = a_{\min} - H = 0.993$. If b is less than this (as for the curve with $b = 0.8$), i.e., if H is too big, there are no solutions. We also note that for $b = 1.2$ in Fig. 9, there is only a solution for small h. This corresponds to supercritical flow; a subcritical solution with larger h is not possible because, at the slow average speed required, the shear would not be compatible with having a positive velocity at the bottom, as assumed in the derivation.

In general the control condition, that a minimum of $J(h;H)$ as a function of h occurs on the h axis, becomes

$$\int_0^Q \frac{g\,\mathrm{d}\psi}{u^3} = 1 \qquad (23)$$

and this may be written as

$$\int_H^{H+h} \frac{g\,\mathrm{d}z}{u^2} = 1. \qquad (24)$$

This is identical to (17) with $c = 0$, implying that long waves are indeed arrested at the control section. Moreover, while (24) implies that the depth-averaged current $\bar{u} \geqslant (gh)^{1/2}$, it also requires that $u = (gh)^{1/2}$ at some point of the profile. This is similar to Stern's (1974) condition for rotating hydraulics, with lateral shear but no vertical shear, that the flow speed at the control section is equal to $(gh)^{1/2}$ at some position across the channel, a result confirmed by Pratt and Armi (1987) from a functional relationship similar to (22) here.

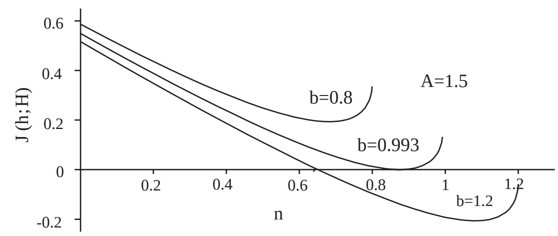

Fig. 9. The function $J(h;H)$ for $a(\psi) = a_{\min} + A\psi$ with $A = 1.5$ and various choices of $b = a_{\min} - H$.

The equivalence of (24) and (17) is, in fact, inevitable once a functional form of the problem has been reached in which a single flow variable (h here) is connected to properties describing the problem. (In this case the bottom height H is the only such property, but the width would enter too if we allowed for it.) Gill (1977) pointed out that if the functional is stationary as a function of the flow variable, then we also have a steady solution for a slight perturbation of that variable, showing that long waves are arrested. It is useful, however, to have the direct proof in this case.

6.4. A general case

It seems that Gill's (1977) functional approach may be stretched to accommodate the case of a sheared flow with a general form of internal friction that does not necessarily maintain the same shape of the velocity profile. We do this by recognising that the Bernoulli constant a in (20) is no longer a function just of the streamfunction ψ but also of x, evolving downstream in response to frictional loss of energy. The variation of a with x is not given a priori, as it is for $H(x)$, but we proceed as if it were. We have already shown that when control occurs, the flow satisfies (24). The location of the control section is given by the condition (Gill, 1977) that J is stationary with respect to changes in H and a with x. The reason for this is that on proceeding downstream the functional curves in a graph such as Fig. 9 must again dip below the h-axis. Using (22), this condition implies that

$$-\frac{\mathrm{d}H}{\mathrm{d}x} + \int_0^Q \frac{\partial a}{\partial x} g u^{-3}\, \mathrm{d}\psi = 0. \tag{25}$$

Now the downstream change $g\,\partial a/\partial x$ of the Bernoulli function $ga(x,\psi)$ is given simply by the right-hand side $\partial\tau/\partial z$ of (12). This may be kept in terms of the vertical coordinate z, so that (25) may be written

$$-\frac{\mathrm{d}H}{\mathrm{d}x} + \int_H^{H+h} \frac{\partial \tau}{\partial z} u^{-2}\, \mathrm{d}z = 0. \tag{26}$$

Integrating the second term by parts and assuming no surface stress and a bottom stress $C_\mathrm{d}u_0^2$, where $u_0 = u(H)$ is the current speed at the bottom, (26) now becomes

$$\frac{\mathrm{d}H}{\mathrm{d}x} = -C_\mathrm{d} + \int_H^{H+h} 2\varepsilon/u^3\, \mathrm{d}z. \tag{27}$$

recognising that the internal dissipation rate ε is given by $\tau(\partial u/\partial z)$, reducing to $\nu(\partial u/\partial z)^2$ if the stress τ is represented as an eddy viscosity ν times the shear.

We see that in the case of no internal dissipation, (27) gives us Pratt's (1986) result for the downstream shift of a control section by bottom friction. The third term in (27) is surprising in that it suggests that internal friction acts in the opposite way to bottom friction; one might have been less surprised to find a factor -1 instead of $+2$, so that the second and third terms could be combined inside an integral having an integrand $(-C_\mathrm{d}u^3 - \varepsilon)/u^3$. This would have paired bottom dissipation with internal dissipation. The reason for the different effects of bottom and internal friction seems to be that the former removes momentum as well as energy from the system, whereas the latter just removes energy. In fact, internal friction acting alone on a flat bottom, without bottom friction, would act to reduce the velocity contribution to the vertically integrated momentum flux $\frac{1}{2}\rho g h^2 + \int_0^h \rho u^2\, \mathrm{d}z$, requiring a compensating downstream increase in layer thickness h. This acts in the opposite sense to bottom friction which requires a decrease in h.

In a sense, therefore, the presence of internal friction opposes bottom friction and counteracts the downstream shift of the control section. A comparison of the magnitudes of the second and third terms in (27) requires specification of the eddy viscosity profile $\nu(z)$ and solution of the governing equations to give $u(z)$. The simplest case is with the specification of a depth-independent eddy viscosity $\nu = Au_0h$, where A is a dimensionless coefficient, and with the assumption that the inertial terms are locally small so that there is a force balance between the internal friction and the pressure gradient. A vertical integral of the balance shows that the pressure gradient then equals the bottom drag $C_\mathrm{d}u_0^2$. The velocity profile is given by

$$u = u_0\left[1 + r\left(\zeta - \tfrac{1}{2}\zeta^2\right)\right], \tag{28}$$

where $r = C_d/A$ and $\zeta = (z - H)/h$. The ratio R of the magnitude of the second term on the right-hand side of (27) to the first term, C_d, is

$$R = \int_0^1 2r(1-\zeta)^2\left[1 + r\left(\zeta - \tfrac{1}{2}\zeta^2\right)\right]^{-3} d\zeta. \qquad (29)$$

This can be expressed analytically as

$$R(r) = \frac{1+r}{2+r} - \frac{2}{r^{1/2}(2+r)^{3/2}} \operatorname{atanh}\left(\frac{r}{2+r}\right)^{1/2} \qquad (30)$$

and is shown in Fig. 10.

For small values of C_d/A, the flow is very viscous internally and so has a slab-like flow and a small value of R. The control section then occurs near where $dH/dx = -C_d$, as in Pratt's (1986) analysis discussed earlier. For large values of C_d/A, the flow is highly sheared, R tends to 1, and the control section moves back to near the crest of the ridge, as for inertial shear flow. One formula for the internal eddy viscosity has $\nu = u_* h/16$, where $u_* = C_d^{1/2} u_0$ is the friction velocity (Csanady, 1982). Thus $A = C_d^{1/2}/16$ and $C_d/A = 16C_d^{1/2}$. With $C_d \simeq 0.005$, as is appropriate for a bottom drag coefficient based on the near-bottom velocity rather than the depth-averaged value, we have $C_d/A \simeq 1.1$ and hence $R \simeq 0.4$, so that the control section occurs where $dH/dx \simeq -0.6C_d \simeq -0.003$. This value of dH/dx is actually close to the result that one would get from Pratt's (1986) formula using a drag coefficient appropriate for use with the depth-averaged, rather than near-bottom, velocity!

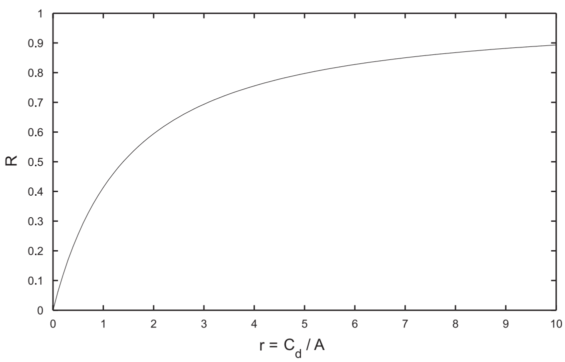

Fig. 10. The function $R(r)$ from (30), giving the ratio of the magnitude of the second term on the right-hand side of (27) to that of the first term.

This analysis assumes a flow that varies sufficiently slowly that the inertial terms are dominated locally by the pressure gradient and frictional forces. If this is not the case, the flow will correspond more closely to that for a purely inertial shear flow, with the control occurring at the crest of the ridge where $dH/dx = 0$.

We also need to note, that, just as in earlier discussion, this development assumes that the internal friction depends only on local flow properties. If the frictional terms depend on downstream derivatives of the flow, their effect in pushing the flow away from being possible is not as simple as, for example, in (5). As we found earlier in discussing the control of a flow with a fixed shape of its velocity profile, frictional terms involving downstream derivatives need to be taken to the left-hand side of the equation of motion and incorporated with the inertial term, changing control conditions and also the long wave speed.

The new approach discussed here, in which the Bernoulli constant is treated as slowly varying under the influence of friction, may also be used in Pratt and Armi's (1987) functional representation of flow, in a rotating channel of rectangular cross-section, with lateral shear but no vertical shear. The current u and layer thickness h are now functions of the cross-channel coordinate y as well as the downstream coordinate x, though the height of the bottom $H(x)$ is still a function of x alone. It can be shown that, for a channel of constant width W, control is shifted downstream of a ridge crest to a location where

$$\frac{dH}{dx}\int_0^W u^{-2}\, dy = -C_d \int_0^W (gh)^{-1}\, dy. \qquad (31)$$

This reduces simply to Pratt's (1986) result $dH/dx = -C_d$ if the channel is narrow or not rotating, so that u and h are independent of y and also $u^2 = gh$ at the control section.

7. Parameterizations

This brief review has so far focussed on general considerations of the effect of friction and entrainment on hydraulic flows in straits. There will certainly be situations, as with short or deep straits, where inertial effects are dominant, though even then, as shown by Bray et al. (1995) for the Strait of Gibraltar, significant interface thickening can occur. In situations where unresolved small-scale processes are important we need to learn how to parameterize them. In this respect, the study of straits raises much the same problems as in many other branches of physical oceanography, and has a particularly large overlap with investigations of estuaries.

7.1. Bottom friction

The parameterization of bottom friction is perhaps the best established with the formula $C_d u^2$ being generally accepted. The extensive literature on appropriate values for C_d will not be discussed here; C_d is generally larger if u is the current speed above some bottom boundary layer, smaller if u is the current speed averaged over the layer depth. I have used 2×10^{-3} as a representative value for the latter case.

7.2. Lateral friction

A topic which seems to have been somewhat ignored is that of lateral mixing. For a homogeneous fluid the small slope of the bottom means that the fluid interior is more likely to be affected by turbulence originating at the sea floor beneath it than farther away laterally. In the presence of stratification, however, the vertical transfer of stress may be greatly inhibited, allowing the influence of lateral eddy momentum transfer from side boundaries. This is likely to be particularly important if the lateral boundary has significant protuberances. In a recent investigation in Juan de Fuca Strait, Colbo (2002) measured the lateral eddy momentum flux within a few kilometres of the coast, finding it equivalent to a lateral eddy viscosity of O(10) $m^2 s^{-1}$, but no general formula in terms of variables such as the tidal current strength is available.

These considerations apply in a situation where rotation is not important. In a rotating system, as discussed earlier and illustrated in Fig. 3, bottom friction can act via the secondary cross-channel flow driven by the bottom Ekman layer. If the channel depth varies laterally, the deceleration will be more pronounced near the shore, giving lateral gradients of the along-channel flow on which lateral mixing can act. Lateral variations in advection, whether low frequency or tidal, can also cause differential density advection and hence lateral density gradients which drive secondary flows across a channel (e.g., Scott, 1994).

While discussing the role of lateral boundaries, it should also be stressed that even channel curvature can also lead to major secondary flows and cause overturning with considerable mixing (Seim and Gregg, 1997).

7.3. Internal friction

There are numerous parameterizations in the literature of the vertical transfer of mass and momentum in a stratified shear flow (e.g., Bowden, 1983). Most assume fluxes proportional to the local gradients, with the eddy viscosity and eddy diffusivity being described in terms of local properties, such as the tidal current which is assumed to be a source of turbulence. Recent models (e.g., Masson and Cummins, 1999) tend to rely on closure schemes such as those of Mellor and Yamada (1982), with justification of the closure relying to some extent on the circumstantial evidence that model predictions of mean flows agree adequately with observations.

Along-strait sea-level gradients can sometimes provide clues to the average internal friction required, at least in situations where the inertial terms are not dominant. For example, in Juan de Fuca Strait, Ott and Garrett (1998) used seasonal differences in sea-level gradient (thus bypassing the levelling problem, given the weak flow in winter) to estimate an interfacial eddy viscosity of up to 0.02 $m^2 s^{-1}$. This value is twice as large as implied by the traditional formula of Bowden and Hamilton (1975) which is based on the rms (mainly tidal) current with a reduction related to the stratification.

There is a dearth of direct observations of eddy fluxes of mass and momentum. For mass, various microstructure techniques (e.g., Wesson and Gregg, 1994) have provided reasonably direct estimates for the eddy diffusivity, but measurements of the eddy viscosity in a stratified shear flow are rare.

Ideally, one would like direct measurements of the vertical eddy momentum fluxes $\overline{u'w'}$ and $\overline{v'w'}$. Acoustic Doppler current profilers (ADCPs) can be used to good effect if the eddies are large enough to be resolved by the bin size and if the eddy statistics are spatially homogeneous so that the beam variance technique can be used (e.g., Lohrmann et al., 1990; Lu and Lueck, 1999). These papers describe Reynolds stress measurement in homogeneous flows with the turbulence generated at the sea floor, whereas we also need data from the stratified shear flows of estuaries and straits. Ott et al. (2002) have been able to determine the Reynolds stresses in Juan de Fuca Strait at various depths below the surface, in water 130 m deep, using a 300 kHz bottom-mounted ADCP sampling with 2 m bins (Fig. 11). The occurrence of significant Reynolds stresses at neap tides coincided with an increase in the mean shear, and decrease in gradient Richardson number, associated with the relaxation of tidal mixing at a constriction upstream of the Strait, thus releasing brackish water from the Strait of Georgia.

Ott et al. (2002) found no simple relationship between the Reynolds stress and the local mean shear, but they did observe secondary flows across the strait that were similar in some respects to those shown in Fig. 4 and expected dynamically. The simplest situation would be a steady response with ageostrophic cross-strait flow v_a given by

$$v_a = f^{-1}\frac{d}{dz}(\overline{u'w'}). \tag{32}$$

This balance was not well satisfied, possibly because advective terms were important, but

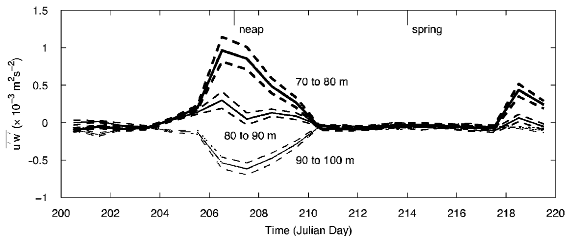

Fig. 11. The daily-averaged Reynolds stress $\overline{u'w'}$ at various depths in the stratified shear flow in Juan de Fuca Strait in 1996. From Ott et al. (2002).

further efforts to determine the Reynolds stresses and the associated ageostrophic secondary flows would seem to be justified. Even without direct Reynolds stress measurements, secondary flows can provide a strong suggestion of internal friction, as in the St. Lawrence estuary (Mertz and Gratton, 1995) or the deep Faroe Channel (Johnson and Sanford, 1992).

8. Conclusions

Much has been learnt about the effect of friction on flows through straits, but the incorporation of frictional effects is still incomplete. On the theoretical side, a key issue is the nature and diagnosis of hydraulic control for flows which have continuous, rather than layered, vertical profiles of density and velocity by virtue of the action of mixing and friction. Although no general results are available, the results discussed in this paper suggest that if mixing and friction depend only on local flow properties, then inviscid long waves are arrested at a control section, though the location of this is shifted. If, on the other hand, mixing and friction depend on downstream derivatives of the flow, as would be required to maintain a fixed shape of the velocity profile in a homogeneous flow, then the control conditions change in line with the changing long-wave speed.

Hydraulic flows, in which flow speeds are comparable with wave speeds, can be complicated, so there is merit to the discussion of idealized situations, where we regard these as valuable building blocks to build intuition and help in the interpretation of observations and the output of complicated numerical models that include many effects. This review has focussed on the role of friction in simple situations, with a major limitation being the assumption of very simple topography such as a rectangular channel. Understanding the effects of irregular topography, even on simple homogeneous flows, is one topic that requires further investigation.

On the observational side, we need more direct measurements of vertical and lateral Reynolds stresses for comparison with parameterization formulae used, rather than arguing that the plausible output of a numerical model justifies the choice of all its ingredients. Observational programs are complicated, of course, by the inhomogeneity on multiple scales of the topography of most real straits, so that choosing "representative" locations for observational programs is a major challenge.

Acknowledgements

I thank Frank Gerdes and Keir Colbo for input, two reviewers and Larry Pratt for helpful comments, and Canada's Natural Sciences and Engineering Research Council and the US Office of Naval Research for financial support. I also thank the organisers of the 2002 Straits Workshop in Villefranche for the invitation to prepare a review on which this paper is based.

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