To:	**OL**
From:	**PJH**
Date:	**September 5, 2002 – 8:30 pm**
Re:	**Boundary Conditions for Barotropic Tidal Velocity**
cc:	**ARR**

Direct application of boundary values to the final barotropic velocity has been shown to be an unmitigated disaster in the non-tidal surface pressure formulation tests. It is possible, however, to make use of barotropic tidal velocity boundary data. The assumption here is that reliable velocity fields can be routinely generated. The two basic issues are: (1) which quantity to impose and (2) where to slip it in.

Barotropic Tidal Velocity versus Tidal Transport

In general, the domain boundaries and topography used for a HOPS PE simulation will not exactly correspond to those of the tidal model being used to generate boundary conditions. The question is: "Is it more important to reproduce the tidal velocities on the boundary of the HOPS domain or the tidal transport?". If the barotropic tidal velocity is chosen, then the boundary values are simply

$$\bar{U}_{PE}\big|_{\partial\mathcal{D}} = \bar{U}_{tide}\big|_{\partial\mathcal{D}} \tag{1}$$

where $\bar{U}_{PE}\big|_{\partial\mathcal{D}}$ are the boundary values for barotropic velocity imposed on the HOPS PE model and $\bar{U}_{tide}\big|_{\partial\mathcal{D}}$ are the barotropic tidal velocities interpolated to the boundary of the HOPS domain.

If the transport is the more important quantity, then the simplest modification is a rescaling,

$$\bar{U}_{PE}\big|_{\partial\mathcal{D}} = \left(\frac{h_{tide}|_{\partial\mathcal{D}} + \eta_{tide}|_{\partial\mathcal{D}}}{h_{PE}|_{\partial\mathcal{D}} + \eta_{tide}|_{\partial\mathcal{D}}}\right)\bar{U}_{tide}\big|_{\partial\mathcal{D}} \tag{2}$$

where $h_{PE}|_{\partial\mathcal{D}}$ is the HOPS topography at the boundary of the domain, $h_{tide}|_{\partial\mathcal{D}}$ is the tidal topography interpolated to the boundary of the domain and $\eta_{tide}|_{\partial\mathcal{D}}$ is the tidal surface elevation interpolated to the boundary of the domain. The tidal elevation is used in both the numerator and denominator since it is directly imposed on the HOPS free surface at the boundary. Note that with multiple tidal components, the real values for velocity and elevation should be computed before the rescaling is applied.

<u>Evaluation</u>

The test for choosing between these two is to try them both in data driven simulation and see which reproduces corresponding gauge data better.

Imposing chosen boundary data

The free surface algorithm employs a time–splitting scheme by defining an augmented velocity, $\hat{\bar{U}}$, as

$$\hat{\bar{U}} = \bar{U}^{n+1} + \alpha g \Delta t \nabla\left(\eta^{n+1} - \eta^{n-1}\right) \tag{3}$$

where α is the implicit parameter from the Coriolis term, g is the acceleration due to gravity, Δt is the time step and the superscripts refer to the time of evaluation. In the course of the simulation, $\hat{\hat{U}}$ is found from

$$(\mathbf{I} + \alpha\Delta t\mathbf{B})\,\hat{\hat{U}} + \alpha g\Delta t\nabla\eta^{n-1} = \vec{F} \tag{4a}$$

where

$$\mathbf{B} = \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \tag{4b}$$

$\mathbf{I}$ is the identity matrix, f is the Coriolis factor and $\vec{F}$ is the vertically integrated right-hand side of the momentum equation. Then the divergence of $\hat{\hat{U}}$ is used to force the equation for surface pressure. After the surface pressure is found, the barotropic velocity is found from (3). The important point is that after the divergence is taken of the augmented velocity, it is no longer safe to try to impose boundary conditions on the barotropic velocity. Therefore any barotropic velocity conditions will have to be imposed on $\hat{\hat{U}}$.

Looking at equation (3) at the boundary, we have available

$\bar{U}^{n+1}\big|_{\partial\mathcal{D}}$ The boundary conditions for the barotropic velocity

$\eta^{n+1}\big|_{\partial\mathcal{D}}$ The boundary conditions for the surface elevation

$\eta^{n-1}\big|_{\partial\mathcal{D}}$ The boundary values for the surface elevation from the previous time step

$\eta^{n-1}\big|_{\partial\mathcal{D}-1}$ The values for the surface elevation from the previous time step at one point inside the boundary.

The values of the surface elevation 1 point inside the boundary are needed to compute the gradient at the velocity boundary. This means that we're missing

$\eta^{n+1}\big|_{\partial\mathcal{D}-1}$ The values for the surface elevation from the next time step at one point inside the boundary

since η^{n+1} is being sought. This means that an approximation is needed. Several such approximations come to mind.

(1) Use the tidal value,

$$\eta^{n+1}\big|_{\partial\mathcal{D}-1} = \eta^{n+1}_{tide}\big|_{\partial\mathcal{D}-1} \quad .$$

The assumption here is that 1 point away from the boundary the boundary values dominate the solution.

(2) Use constant extrapolation to correct the tidal value,

$$\eta^{n+1}\big|_{\partial\mathcal{D}-1} = \eta^{n+1}_{tide}\big|_{\partial\mathcal{D}-1} + \delta\eta \quad .$$

Some choices would be

$$\begin{aligned}
\delta\eta &= \Delta\eta^{n} \equiv \eta^{n}\big|_{\partial\mathcal{D}-1} - \eta^{n}_{tide}\big|_{\partial\mathcal{D}-1} \\
\delta\eta &= \Delta\eta^{n-1} \\
\delta\eta &= \frac{1}{2}\left(\Delta\eta^{n} + \Delta\eta^{n-1}\right)
\end{aligned}$$

(3) Use linear extrapolation to correct the tidal value. Using the same notation as in the constant extrapolation case,

$$\delta\eta = \Delta\eta^n + \left(\Delta\eta^n - \Delta\eta^{n-1}\right)$$

(4) Use the change of surface elevation data. At any time prior to solving the elliptic equation for surface pressure, the model has in storage

$$\partial_t\eta^n \equiv \eta^n - \eta^{n-2}$$

This information could also be used to generate corrections

$$\begin{aligned}
\partial_t\eta^{n+1}\big|_{\partial\mathcal{D}-1} &= \partial_t\eta^n|_{\partial\mathcal{D}-1} \\
\partial_t\eta^{n+1}\big|_{\partial\mathcal{D}-1} &= \partial_t\eta^{n+1}_{tide}\big|_{\partial\mathcal{D}-1} + \left(\partial_t\eta^n|_{\partial\mathcal{D}-1} - \partial_t\eta^n_{tide}|_{\partial\mathcal{D}-1}\right)
\end{aligned}$$

<u>Evaluation</u>

The issue here is that boundary conditions for the barotropic velocity, $\bar{U}$ are used to generate approximate boundary conditions for the augmented velocity, $\hat{\bar{U}}$. In the course of solution the barotropic velocity is regenerated at the boundary. The test will be how well these regenerate values match those imposed.