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Quick summary solution of PS 2 of Recitation 4, 2.006. Spring 2007.

A] For no mass transfer, at $r=R$, $v_r = \dot{R}(t)$

B] $\phi = \phi(r \text{ only})$ $\nabla^2\phi = 0 \iff \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) = 0$

spherical coordinates

Integrating the equation, one gets: • $r^2\frac{\partial\phi}{\partial r} = 1 \Rightarrow \frac{\partial\phi}{\partial r} = \frac{c_1}{r^2} \equiv v_r$

• $\phi = -\frac{c_1}{r} + c_2$

BC: • at $r=R$, $v_r = \dot{R} \equiv \left.\frac{\partial\phi}{\partial r}\right|_{r=R} \Rightarrow c_1 = R^2\dot{R}$

• $c_2 \equiv 0$ (arbitrary)

$\Rightarrow \phi = -\frac{R^2\dot{R}}{r}$

C] Velocity distribution

$\underline{u} = \nabla\phi = \frac{\partial\phi}{\partial r}\underline{e}_r + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\underline{e}_\theta \Rightarrow \begin{cases} u_r = \frac{\partial\phi}{\partial r} = +\frac{R^2\dot{R}}{r^2} \\ u_\theta = 0 \end{cases}$

check: at $r=R$, $u_r = \dot{R}$

D] Pressure inside the bubble, use Bernoulli

At ∞: $p = p_\infty$, $u_r = 0$

$\frac{\partial\phi}{\partial t} + \frac{p}{\rho} + \frac{v^2}{2} + g\cancel{z} = \text{cte}$ (neglect)

$\Rightarrow \text{cte} = \underbrace{\left.\frac{\partial\phi}{\partial t}\right|_\infty}_{=0} + \frac{p_\infty}{\rho} + 0 = \frac{p_\infty}{\rho}$

(i) $\frac{\partial\phi}{\partial t} = \frac{-2R\dot{R}^2 - R^2\ddot{R}}{r}$

(ii) $v^2 = u_r^2 = \frac{R^4\dot{R}^2}{r^4}$

$\Rightarrow p = \rho\left[\frac{2R\dot{R}^2 + R^2\ddot{R}}{r} - \frac{R^4\dot{R}^2}{r^4}\right] + p_\infty$

at $r=R$, $p = \rho\left[2\dot{R}^2 + R\ddot{R} - \dot{R}^2\right] + p_\infty$

$$\Rightarrow \boxed{p_{r=R} = p_\infty + \rho\left[\dot{R}^2 + R\ddot{R}\right]}$$