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Adaptive Sampling Using Fleets of Underwater Gliders in the Presence of Fixed Buoys using a Constrained Clustering Algorithm

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***Abstract*—This paper presents a novel way to approach the problem of how to adaptively sample the ocean using fleets of underwater gliders. The technique is particularly suited for those situations where the covariance of the field to sample is unknown or unreliable but some information on the variance is known. The proposed algorithm, which is a variant of the well-known fuzzy *C*-means clustering algorithm, is able to exploit the presence of non-maneuverable assets, such as fixed buoys. We modified the fuzzy *C*-means optimization problem statement by including additional constraints. Then we provided an algorithmic solution to the new, constrained problem.**

***Index Terms*—adaptive sampling, underwater gliders, clustering algorithms, fuzzy *C*-means.**

I. Introduction

The use of fleets of underwater gliders to sample the ocean has been proven to be an appealing alternative to more expensive solutions such as those based on vessels [1][[2][[3][4][[5][[6]. Because of the long endurance of the mission of a fleet of gliders (order of months), when compared to the endurance of the mission of AUVs (order of days), gliders are nowadays routinely involved in ocean sampling campaigns [7][8][9][10][11][12]. However, where to direct the vehicles (the so-called "adaptive sampling" problem) in order to maximize the content of information gathered is still an active research topic [13][14][15].

The underlying philosophy of some adaptive sampling algorithms is the following. First, the knowledge of the a-priori covariance matrix of the considered discretized field (temperature, salinity, etc.) is assumed to be available (either provided by a forecasting model or estimated from forecasts/analyses/reanalyses) for the region of interest. Then the adaptive sampling algorithm generates a list of waypoints, using an assimilation algorithm to estimate the posterior covariance matrix and optimize the reduction of a norm of this covariance (e.g. [7]). We will refer to this class of algorithms as *covariance-based* adaptive samplers with assimilation.

However in some cases the a-priori covariance matrix cannot be used, because it is not available at all or it is available but judged not enough reliable (sometimes this can be due simply to the fact that it has not been validated yet). In such cases a viable alternative (although less powerful) is represented by the use of the variance of the field (e.g. [16][17][18]), instead of its covariance.

The schemes based on the variance may use or not an assimilation phase. Of course, in case the assimilation is present, it will involve the variance only, in order to estimate the posterior variance. We will call these approaches as *variance-based* adaptive samplers with/without assimilation.

In this work we present a new adaptive sampling algorithm, specifically designed for the sampling missions where: i) the covariance matrix is not available/not reliable/not validated, and ii) non-maneuverable sampling assets are present (such as fixed buoys, drifters, and floating buoys), in addition to maneuverable ones (gliders, AUVs, etc.). In this scenario, since the a-priori covariance matrix is unknown, it cannot be used to assimilate the measurements of the fixed buoys before planning the mission of the gliders. Instead, we have to decide where to direct the gliders (i.e., to generate their lists of waypoints), exploiting the presence of fixed buoys.

The algorithm extends an earlier version presented in [19], which was based on the idea of deriving a distribution of points from the a-priori variance (by using an adaptive meshing algorithm) and then using a clustering algorithm to find the location of the centroids, where such locations were adopted as next waypoints for the gliders.

However, that method assumed that all the assets where maneuverable (and thus the number of centroids was equal to the number of gliders). In this study we extend it by exploiting the existence of non-maneuverable assets such as fixed buoys (a situation that frequently occurs in real scenarios).

The first essential idea is to consider the positions of fixed buoys as a subset of the centroids obtained though the clustering algorithm: the remaining centroids to be computed will be considered as the next positions where to direct each glider. By using the clustering algorithm described in [20], called "Partially Provided Centroids Fuzzy *C*-Means" (ppcFCM), we have been able to exploit the presence of fixed buoys by directing the gliders in regions not already covered by them. This allows a better distribution (lower overlapping) of the sensing assets, with respect to the direct use of the standard Fuzzy *C*-Means (FCM), uninformed of the presence of the buoys.

An interesting advantage of the proposed algorithm over covariance-based schemes with assimilation, is that it can be used in surveillance applications too, where a time variant risk map (generated by other tools, e.g. [21][22][23][24]) is assumed to be available, together with the presence of some fixed observing stations (harbor control towers, coast guard station, etc.).

II. BACKGROUND: ADAPTIVE SAMPLING USING CLUSTERING

Underwater gliders are robotic vehicles able to collect measurements (temperature, salinity, etc.) while following an up and down, saw-tooth profile through the water. This up and down movement is repeated until a given amount of time (ΔT) has elapsed. Then the gliders go up to the surface and stay there for a while (another pre-programmed amount of time, while the vehicle sends the collected data back to a ground station and waits for the new mission parameters). In particular, each glider receives its next waypoint, i.e., the new location where to surface, after the next ΔT.

The ocean adaptive sampling problem using underwater gliders consists in determining where to direct them (e.g., the corresponding list of waypoints) in order to collect the most important measurements. This approach has been shown to be more efficient than a brute force approach based on the uniform sampling on a regular grid covering the area to sample. Typically, the waypoints are chosen to reduce the predicted uncertainties in ocean forecast products. When this uncertainty is not available, operational oceanographers typically consider the variability in the ocean forecasts (such as those that can be downloaded from the MyOcean repository [25]). When the covariance of the field to sample (temperature, salinity, etc.) is known, more powerful techniques can be used (e.g. [7][26][27][28][29]). On the contrary, when the covariance is unavailable, a technique like the one based on clustering and presented in [19] can be used. We will review it in next subsection.

A. Adaptive sampling using Fuzzy C-Means

Consider a fleet of underwater vehicles already deployed underwater. Suppose the gliders are programmed to surface simultaneously, every ΔT. When they reach the surface, they wait for the next waypoint. The strategy proposed in [19] is the following:

- Step 1: Obtain an uncertainty/variability distribution for next time frame
- Step 2: Mesh it, in order to obtain a point cloud distribution
- Step 3: Run a clustering algorithm, in order to estimate the new centroids
- Step 4: Send each vehicle to its closest centroid, among the one computed in previous step
- Step 5: repeat from step 1, until the mission ends.

In what follows, we discuss some of these steps in more detail.

B. More details about step 1

Certain ocean field forecast providers not only supply the forecast but also the associated uncertainty [10][30][31]. In particular, if an ensemble model is used for generating the forecasts, then the variance-based uncertainty is straightforwardly available as the standard deviation between multiple model runs. On the contrary, when a single model is used, the uncertainty associated with the forecast is not frequently available. In this case, we can consider the *variability* in the forecasts (i.e., the standard deviation of multiple forecasts of the hours/days ahead). The idea is to direct the sampling assets in regions where the variability is higher, assuming that there the most important phenomena are occurring. One way to obtain the variability map of temperature or salinity is to resort to the MyOcean service (http://www.myocean.eu), following the approach described in [25]: once the forecasts have been downloaded (e.g., for a sub-region of the Mediterranean Sea, 10 days ahead), the standard deviation among the 10 days can be computed and used as variability map. Then, after 3/4 days (the re-planning period), the new forecasts (which are recomputed every day) can be downloaded and the new variability recomputed. Additional links between variability and uncertainty are discussed in [32][33].

C. More details about step 2

Adaptive meshing is routinely used in Finite Elements methods to influence the density of the mesh depending on an error function/potential function. It allows computing a (triangular) mesh having a density of triangles proportional to a given field, as shown in Fig. 1.

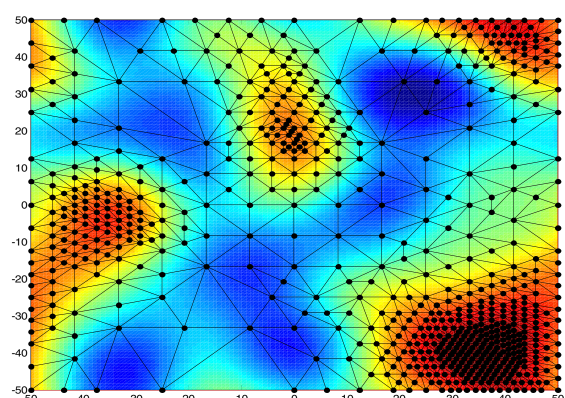

Fig. 1. The adaptive triangular mesh computed from a given uncertainty field.

Then, by discarding the information about the edges, we obtain a pure point cloud distribution such as the one in Fig. 2. The clear advantage of having transformed an error function into a point cloud distribution resides in the fact that it can be easily handled by most clustering algorithms.

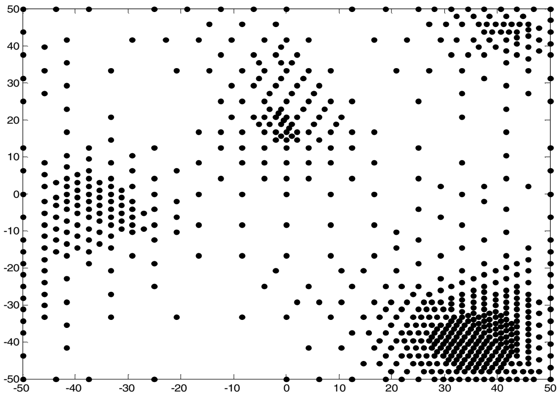

Fig. 2. The point cloud distribution obtained after discarding edges.

D. More details about step 3: the FCM clustering algorithm

In step 3 a clustering algorithm is needed to locate the centroids of the point cloud. The number of centroids is set equal to the number of maneuverable assets (the yellow diamonds in Fig. 3).

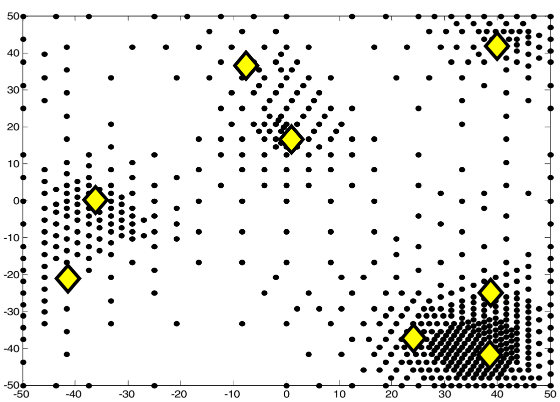

Fig. 3. The centroids computed by FCM algorithm, corresponding to the next waypoints of the gliders.

The FCM algorithm [34][35][36] computes the position of the C centroids by minimizing the following cost function: $J_m^C = \sum_{d=1}^{D}\sum_{c=1}^{C} u_{dc}^m \left\| \boldsymbol{\delta}_d - \boldsymbol{\gamma}_c \right\|^2$, where: $\{\boldsymbol{\delta}_d\}_{d=1}^D$ are the D points in an F-dimensional space ($\boldsymbol{\delta}_d \in \Re^{1\times F}$) to clusterize; $\{\boldsymbol{\gamma}_c\}_{c=1}^C$ are the C unknown centroids to compute (again, $\boldsymbol{\gamma}_d \in \Re^{1\times F}$); u_{dc} are the fuzzy membership degrees; and m is the fuzziness exponent (usually, m=2 is used). In particular, u_{dc} (which ranges between 0 and 1) indicates how much the d^{th} point belongs to the cluster represented by c^{th} centroid. All the u_{dc} membership degrees (which are constrained to sum up to one for any given value of d) can be organized into the matrix $\mathbf{U} = \{u_{dc}\} \in \Re^{C\times D}$.

It is convenient to organize row by row also the points and the centroids into two matrices, $\boldsymbol{\Delta} \in \Re^{D\times F}$ and $\boldsymbol{\Gamma} \in \Re^{C\times F}$, respectively. The FCM algorithm works as explained in Algorithm 1. Basically, after the initialization, it repeats for I times a core iteration which first computes the new centroids ${}^{i}\boldsymbol{\Gamma}$ from previous membership degrees ${}^{(i-1)}\mathbf{U}$, and then the new membership ${}^{i}\mathbf{U}$ degrees from current centroids ${}^{i}\boldsymbol{\Gamma}$.

Algorithm 1: Standard FCM Algorithm

Input: $\boldsymbol{\Delta}$, C, m, I **Output:** $\mathbf{U}$, $\boldsymbol{\Gamma}$

1 Initialize matrix ${}^{(0)}\mathbf{U}$ at random.
for i = 1 to I **do**

2 $${}^{i}\gamma_c = \frac{\sum_{d=1}^{D} {}^{(i-1)}u_{dc}^m \delta_d}{\sum_{d=1}^{D} {}^{(i-1)}u_{dc}^m}, \quad \forall c = 1 \ldots C$$

3 $${}^{i}u_{dc} = \frac{1}{\sum_{k=1}^{C}\left(\frac{\left\|\boldsymbol{\delta}_d - {}^{i}\boldsymbol{\gamma}_c\right\|}{\left\|\boldsymbol{\delta}_d - {}^{i}\boldsymbol{\gamma}_k\right\|}\right)^{\frac{2}{m-1}}}, \forall d, \forall c$$

III. Adaptive Sampling Using Clustering with Both Fixed and Mobile Assets

In real sampling scenarios it can happen that within the sampling area we have non-maneuverable assets, such as fixed buoys. Ignoring the presence of these assets can lead to suboptimal assignments, since the presence of fixed buoys is not exploited by the above FCM algorithm. Thus the new problem statement is the following:

> Considering a sampling area A, a set of maneuverable assets (E gliders) and a set of non-maneuverable assets (P fixed buoys), determine the next waypoints of the maneuverable assets exploiting the presence of non-maneuverable ones.

As in Section II, we can define a cost function involving all the P+E=C assets, where, however, this time we have some of the centroids that must be initialized with the position of the non-maneuverable assets and then must remain constrained to that value for all the time. Only the centroids associated to the E gliders can be optimized in order to minimize the new cost function $J_m^{C(P)}$:

$$\begin{cases} J_m^{C(P)} = \sum_{d=1}^{D}\sum_{c=1}^{C} u_{ij}^m \left\|\boldsymbol{\delta}_d - \boldsymbol{\gamma}_c\right\|^2 \\ \textit{subject to}: \\ \boldsymbol{\gamma}_c = \boldsymbol{\pi}_c, \ \forall c = 1 \ldots P \quad (\textit{with } P < C) \end{cases} \tag{1}$$

This means that part of the centroids is known in advance, while the other part has to be found. The P provided centroids can be organized within matrix $\mathbf{\Pi} \in \Re^{P \times F} = \begin{bmatrix} \boldsymbol{\pi}_1 \\ \dots \\ \boldsymbol{\pi}_P \end{bmatrix}$.

Therefore the whole centroid matrix $\mathbf{\Gamma}$ can be expressed as:

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Pi} \\ \mathbf{\Sigma} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\gamma}_1 \\ \dots \\ \boldsymbol{\gamma}_P \\ \boldsymbol{\gamma}_{P+1} \\ \dots \\ \boldsymbol{\gamma}_C \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}a\\a\\a\end{matrix}}\right\} \textit{the P provided centroids } (\mathbf{\Pi}) \\ \left.\vphantom{\begin{matrix}a\\a\\a\end{matrix}}\right\} \textit{the } C-P=E \textit{ centroids to be computed} \end{matrix}$$

The problem described above can be solved algorithmically, as explained in next section.

IV. An algorithmic solution

A suitable way to solve problem (1) is to adapt the FCM algorithm summarized in Section II in order to account for the constraints.

This algorithm, referred to as Partially Provided Centroids FCM (ppcFCM) is a variant of the FCM that takes as input both the data and the partial list of P centroids (in our case, the position of the P fixed buoys). It then computes the position of the free centroids, in order to minimize $J_m^{C(P)}$, i.e., the cost function associated to the use of C centroids, P of them being fixed. This Algorithm 2 excerpt provided below outlines the ppcFCM algorithm.

Algorithm 2: ppcFCM

Input: $\mathbf{\Delta}$, C, m, I, $\mathbf{\Pi}$ **Output:** $\mathbf{U}$, $\mathbf{\Gamma}$

1 Set the initial centroids ${}^{(0)}\boldsymbol{\gamma}_c$ equal to $\boldsymbol{\pi}_c$, $\forall c = 1 \dots P$ and the remaining (C-P=E) centroids at random.

for $i = 1$ to I **do**

2 $${}^{i}u_{dc} = \frac{1}{\sum_{k=1}^{C} \left(\frac{\left\| \boldsymbol{\delta}_d - {}^{(i-1)}\boldsymbol{\gamma}_c \right\|}{\left\| \boldsymbol{\delta}_d - {}^{(i-1)}\boldsymbol{\gamma}_k \right\|} \right)^{\frac{2}{m-1}}}, \ \forall d, \forall c$$

3 $${}^{i}\gamma_c = \frac{\sum_{d=1}^{D} {}^{i}u_{dc}^{m} \boldsymbol{\delta}_d}{\sum_{d=1}^{D} {}^{i}u_{dc}^{m}}, \quad \forall c = (P+1) \dots C$$

The two algorithms (FCM and ppcFCM) differ in the initialization (step 0) and in the order of the two steps within the loop. Furthermore, they differ in the step that updates the centroids (step 2 in FCM and step 3 in ppcFCM), since in ppcFCM the updating formula is used only for the free centroids (thus, obviously, the other will remain unchanged throughout the whole optimization process). More details about the ppcFCM algorithm can be found in [20]. Two properties of this algorithm are particularly noteworthy: i) it fulfills the constraints by construction, ii) it can be sped up by exploiting the fact that part of the centroids is known at the beginning and do not change with time. Thus, when C is the same for FCM and ppcFCM, the latter can be made faster than the former by reusing part of the computations (more details on this aspect can be found in [20]).

V. Results

On a simulated scenario, we have compared the behavior of the standard FCM algorithm (uninformed of the presence of the fixed buoys) with ppcFCM. We have assumed to have a fleet of E=5 gliders and to have to sample a rectangular oceanic region where P=3 fixed buoys are present (gray spots in next figure). We have simulated an uncertainty field made by a mixture of Gaussian distributions. Then we have run a triangular meshing algorithm, able to refine the mesh were the uncertainty is higher (see Fig. 4).

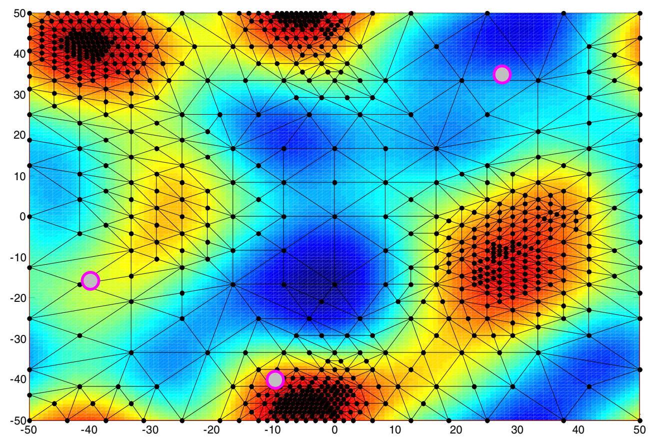

Fig. 4. The simulated uncertainty distribution, the computed mesh and the position of the fixed buoys (gray and magenta circles).

Figure 5 shows the centroids (i.e., next waypoints), computed by the standard FCM algorithm with C=E=5 (the number of gliders) after 200 iterations (I = 200).

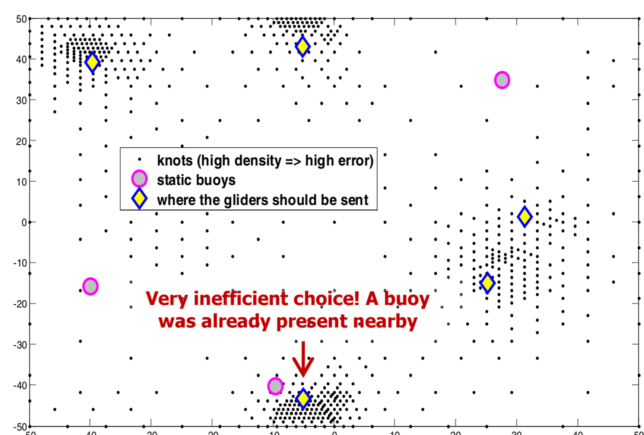

Fig. 5. The centroids (yellow diamonds) computed by the FCM, uninformed of the presence of the fixed buoys.

As we can see, one of the gliders has been directed to a region where a buoy is already present, thus affecting the effectiveness of the sampling.

Figure 6 shows the waypoints computed by our ppcFCM, run with *C*=8 centroids, *P*=3 of which are constrained to be where the buoys are.

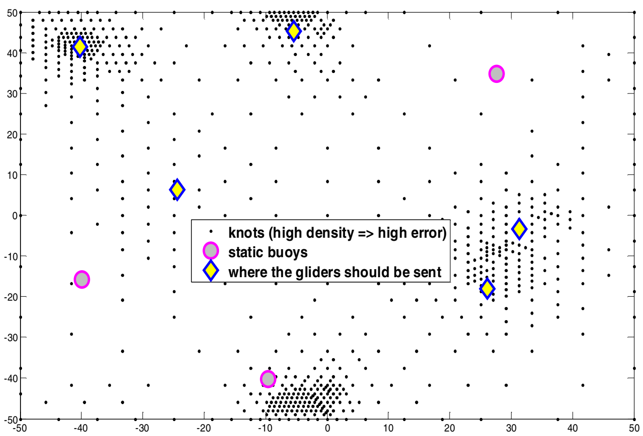

Fig. 6. The centroids (yellow diamonds) computed by the ppcFCM, informed of the presence of the fixed buoys.

As we can see, now the 5 gliders have been directed away from where a buoy is present, but still where the point density is higher. This better solution is confirmed by the fact that the cost function $J_m^{8(3)}$ is lower than its FCM counterpart J_m^5 : $J_m^{8(3)}$ =55735, while $J_m^5 = 86453$.

VI. Conclusions

We proposed the application of a clustering algorithm to the adaptive sampling problem in the presence of both maneuverable and non-maneuverable assets. The algorithm is able to exploit the presence of non-maneuverable assets (such as fixed buoys), by considering them as constrained centroids. The remaining (free) centroids are optimized in order to minimize a cost function. Once the free centroids have been estimated, they are used as next waypoints for the maneuverable assets. The clustering algorithm proposed is able to fulfill the constraints by construction, in an elegant and effective way. As a final note, we remark how the algorithm could be easily adapted to situations where the gliders go to the surface asynchronously, and/or to the case of moving less-maneuverable or non-maneuverable assets, such as drifters. We are now considering to further speedup the algorithm by using GP-GPU [37].

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