Autonomous Path Planning to Optimally Harvest Dynamic Fields

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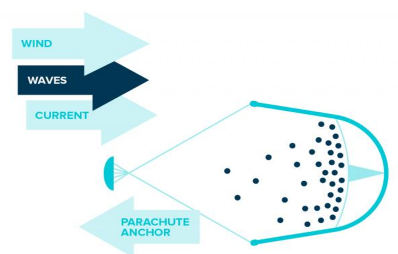
Background and Motivation

The optimal collection and monitoring of active materials is essential for the efficient protection and sustainable utilization of the ocean.

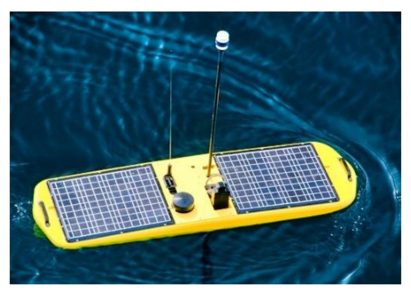
Examples of problems we aim to solve

- Time-optimal collection of a field of dynamic marine material
- Time-optimal path planning for dynamically-constrained vehicles that can collect energy from their surroundings


Marine Plastic Pollution



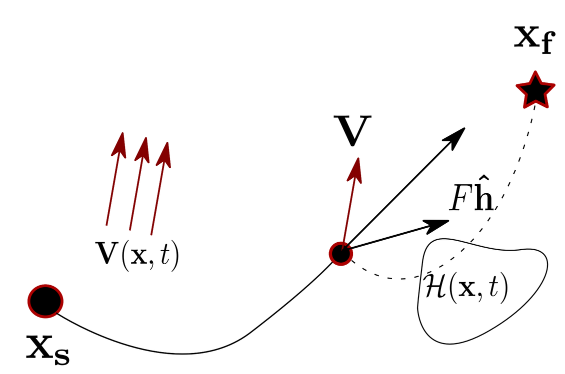
The Ocean Cleanup


Liquid Robotics

Objective and Problem Statement

Find the quickest path for a vehicle:

- From start point to destination
- In a dynamic flow field in which the vehicle is being advected
- While collecting a required amount of background field, which is also dynamic!



Problem Parameters

- Background flow $\mathbf{V}(\mathbf{x},t)$
- Max vehicle speed F_{max}
- Vehicle heading $\hat{\mathbf{h}}$
- Vehicle dynamics $\frac{d\mathbf{x}}{dt} = F\hat{\mathbf{h}} + \mathbf{V}(\mathbf{x},t)$
- Start $\mathbf{x}_s$
- Destination $\mathbf{x}_d$
- Harvesting Dynamics $\frac{d\mathbf{c}}{dt} = \mathcal{H}(\mathbf{x},\mathbf{c},F,\hat{\mathbf{h}},t)$

Current state of the art methods for time-optimal path planning:

- Rapidly exploring random trees
- A^* search
- Artificial potential methods

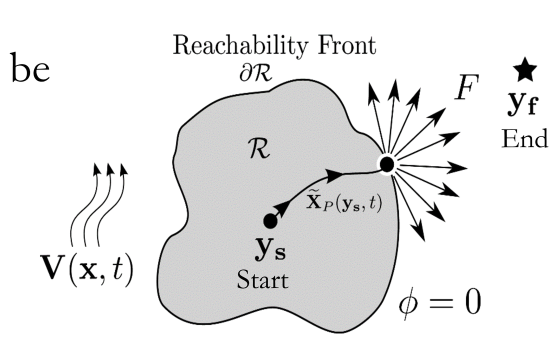
Issues: sub-optimal, heuristic dependent, for static fields

We base our method on the algorithm developed for time-optimal path planning using the Level Set method – An exact equation which applies in any strongly dynamic background flows

Time Optimal Path Planning with Level Sets

Reachability front

- Contour containing the set of positions that can be reached at a given time
- Implicitly represented as the zero level set of a signed distance function ϕ
- The function ϕ physically corresponds to the following cases:
  - $\phi(\mathbf{x},t) < 0$: Point reachable in time t
  - $\phi(\mathbf{x},t) = 0$: Point on reachability front at time t
  - $\phi(\mathbf{x},t) > 0$: Point not reachable in time t



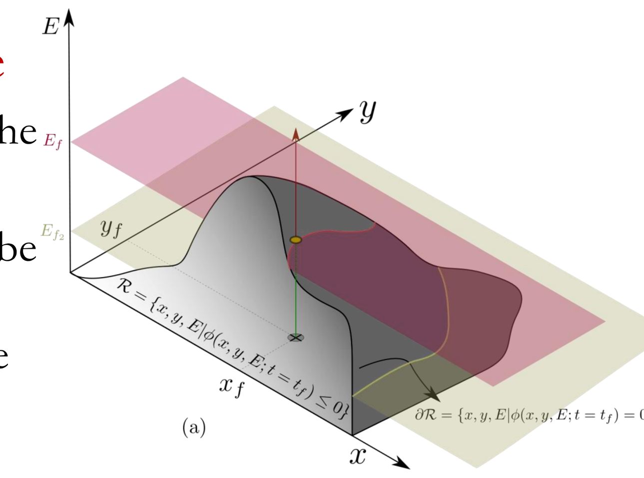
Start: $\phi(\mathbf{x},0) = |\mathbf{x} - \mathbf{x}_s|$

End: $\phi(\mathbf{x}_f, T^o(\mathbf{x}_f)) = 0$

Harvest-Time Optimal Path Planning

Reachability front in augmented state space

- Consider and augmented state space containing the vehicle position and harvest field (e.g., energy).
- Reachability set is now the set of **states** that can be reached at a given time. If a vehicle can reach a certain point **at the prescribed energy level**, the point in the state space (the position and energy level) is in the reachable set



Key steps for computing forward level set evolution

Specify **all** state of interest and determine their dynamics ➡ Determine the optimal dynamics for reachability front evolution – Maximize normal excursion from the level set ➡ Create a PDE whose characteristics are the optimal dynamics and thus governs the implicit function (ϕ) dynamics

Time optimal path planning

$$\frac{d\mathbf{x}}{dt} = F\hat{\mathbf{h}} + \mathbf{u}(x,t) \Rightarrow \frac{d\mathbf{x}}{dt} = V + \max\left(F\hat{h}\cdot\hat{n}\right) \Rightarrow \frac{\partial\phi}{\partial t} + F_{max}|\nabla\phi| + \mathbf{V}\cdot\nabla\phi = 0$$

Harvest-time optimal path planning

$$\text{State Space} = \begin{bmatrix}\text{Position}\\ \text{Amount Collected}\end{bmatrix}$$

$$\frac{d\mathbf{x}}{dt} = F\hat{\mathbf{h}} + \mathbf{V}(x,t), \quad \frac{d\mathbf{c}}{dt} = \mathcal{H}(\mathbf{x},\mathbf{c},F,\hat{h},t) \Rightarrow \frac{d\begin{bmatrix}\mathbf{x}\\ \mathbf{c}\end{bmatrix}}{dt} = \begin{bmatrix}\mathbf{V}(\mathbf{x},t)\\ 0\end{bmatrix} + \max\left(\begin{bmatrix}F\hat{h}\\ \mathcal{H}(\mathbf{x},\mathbf{c},F,\hat{h},t)\end{bmatrix}\cdot\hat{n}\right)$$

$$\frac{\partial\phi}{\partial t} + \mathbf{V}\cdot\nabla_x\phi + \max\left(F\hat{\mathbf{h}}\cdot\nabla_x\phi + \mathcal{H}\cdot\nabla_c\phi\right) = 0$$

Maximization for Specific Cases

Optimal Collection Path Planning

$$\frac{dc}{dt} = \alpha C(\mathbf{x}(t),t)$$

$$\hat{\mathbf{h}}^* = \frac{\nabla_x\phi}{|\nabla_x\phi|} \qquad F^* = F_{max}$$

$$\frac{\partial\phi}{\partial t} + \mathbf{u}\cdot\nabla_x\phi + \alpha C\frac{\partial\phi}{\partial c} + F_{max}|\nabla_x\phi| = 0$$

TVD RK4 | 5[th]-order WENO | Godunov Method

Energy Optimal Path Planning

$$\frac{dc}{dt} = \mathcal{H}(\mathbf{x},F,\hat{\mathbf{h}},t) = -kF^3 + \dot{Q}(x,t)$$

$$\hat{\mathbf{h}}^* = \frac{\nabla_x\phi}{|\nabla_x\phi|} \qquad F^* = \begin{cases}\min\left(F_{max}, \sqrt{\frac{|\nabla_x\phi|}{3k\frac{\partial\phi}{\partial c}}}\right) & \frac{\partial\phi}{\partial c} > 0\\ F_{max} & \frac{\partial\phi}{\partial c} < 0\end{cases}$$

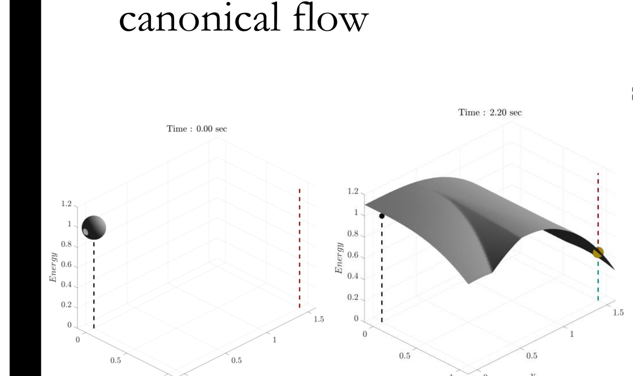
$$\frac{\partial\phi}{\partial t} + \mathbf{u}\cdot\nabla_x\phi + \left(-kF^{*3} + \dot{Q}\right)\frac{\partial\phi}{\partial c} + F^*|\nabla_x\phi| = 0$$

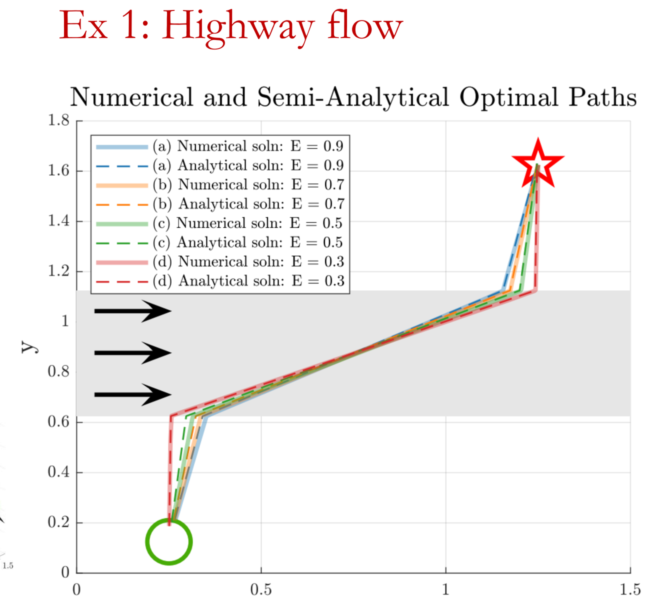
TVD RK4 | 5[th]-order WENO | Godunov Method

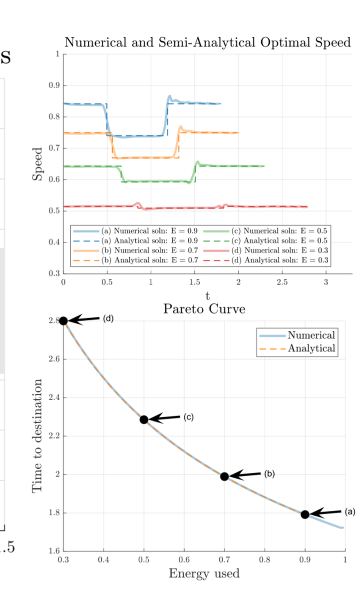
Validation: Energy Optimal Path Planning

We compare the solutions obtained using our methodology to semi-analytical solutions in a canonical flow

Ex 1: Highway flow



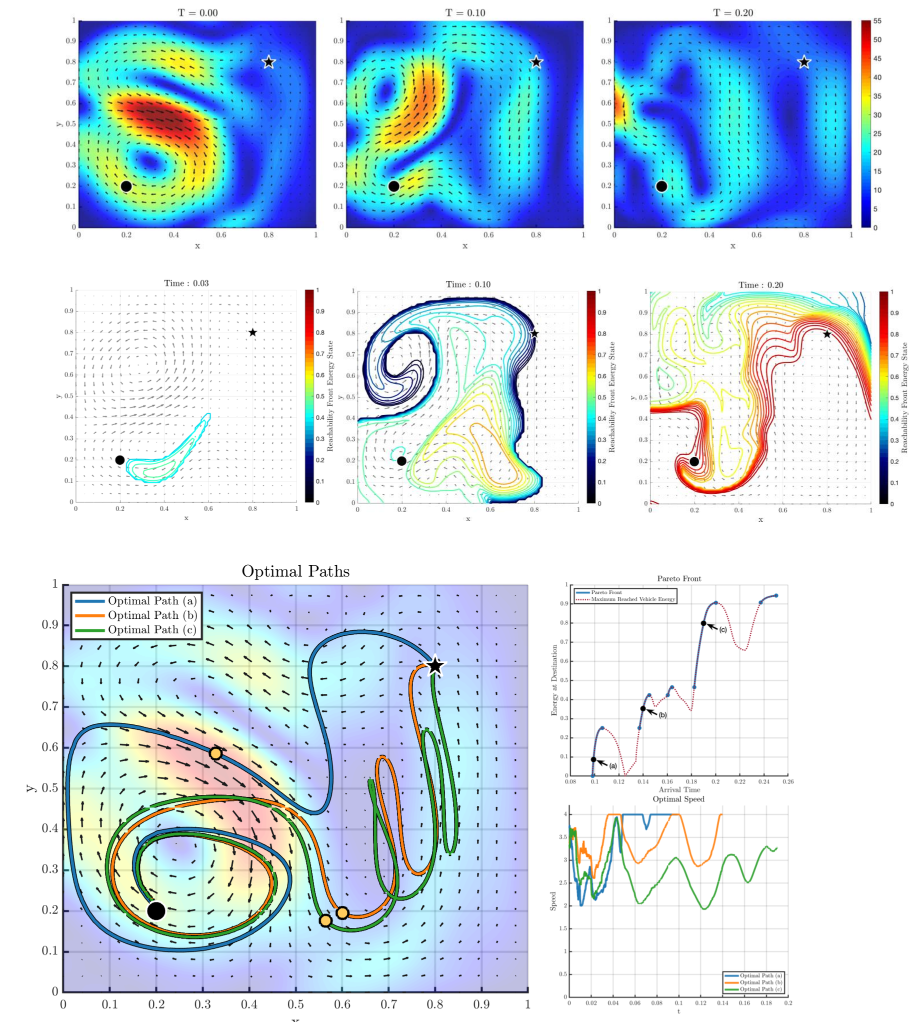




Results

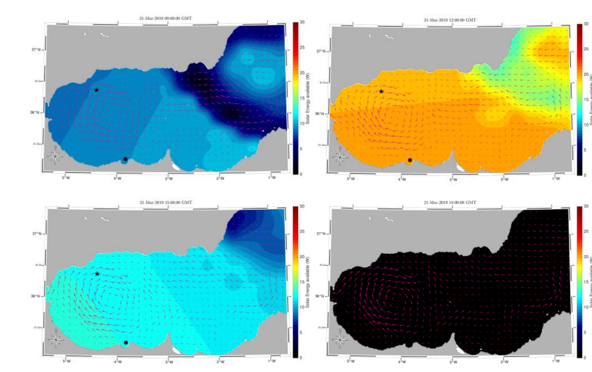
Ex 2: Energy optimal path planning in a quasi-geostrophic double gyre flow

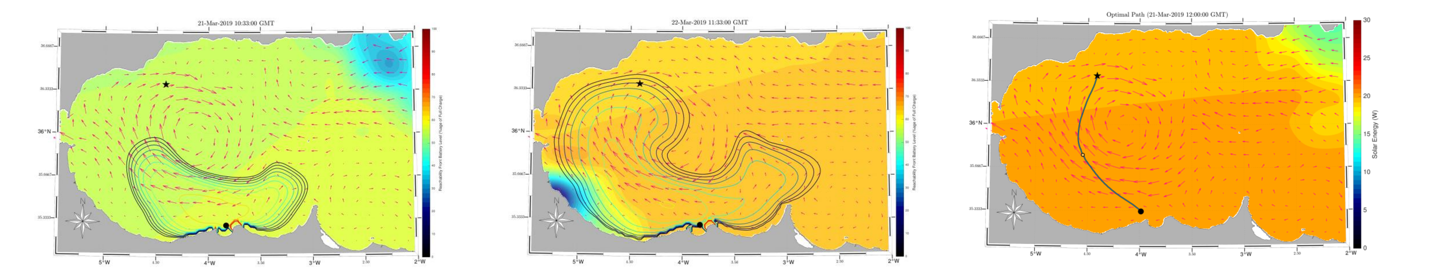
- A quasi-geostrophic double gyre flow shown on the right simulates near surface ocean circulation at mid-latitude regions.
- We consider the case where the vehicle can recharge batteries using solar radiation available in the southeast corner of the domain.
- In a strong flow, the reachability front is very strongly governed by the background currents
- The optimal paths obtained for various energy constraints are highly non-trivial



Ex 3: Realistic simulation with parameters for SAUV II

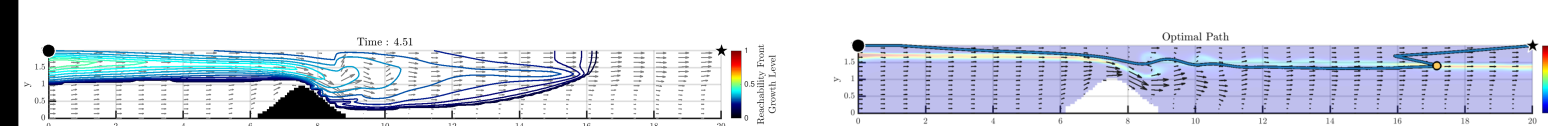
- We now test this algorithm with realistic vehicle parameters in the Mediterranean region. The ocean forecasts and the solar radiation forecasts are shown to the right. Shown below are the plots for the level set evolution and the final optimal path obtained



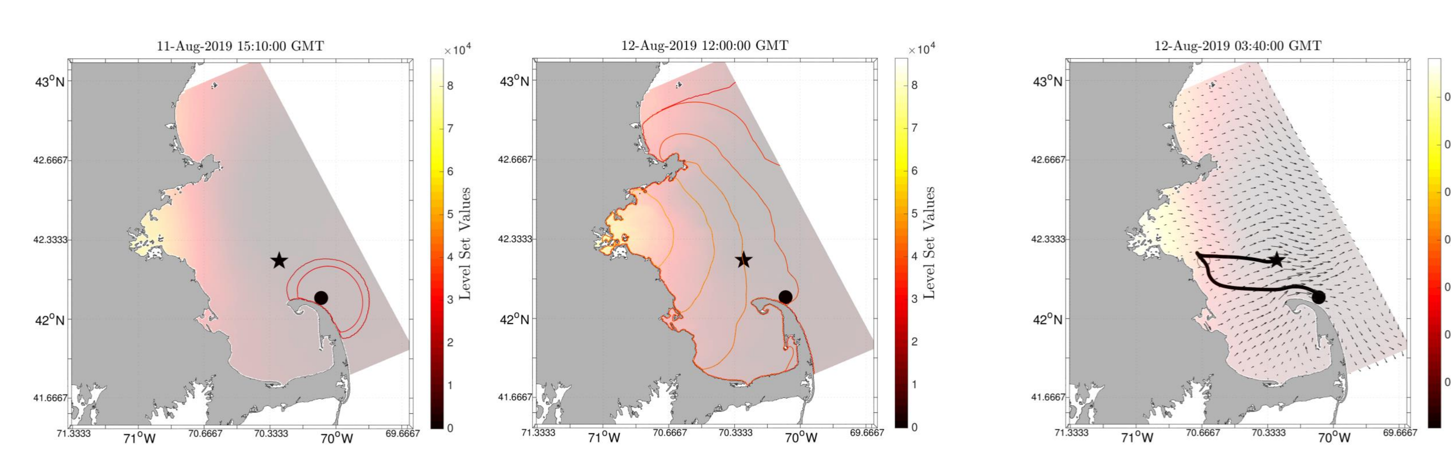


Ex 4/5: Proof of concepts: Moving fish farms and Kelp harvesters

- Other potential application of this algorithm is in moving fish farms. Nutrient densities for fish can vary drastically with depth. By modelling the nutrients, we can find the optimal path that a fish farm should follow to allow the fishes to multiply rapidly in the minimum possible time.



- By modelling kelp growth, we can also plan optimal paths for harvesters, allowing them to operate efficiently and collect the most amount of kelp in the least amount of time.



Conclusions and Future Work

- Derived exact equation and algorithm for time-optimal collection of background field by extending our existing equations for time-optimal path planning
- Demonstrated successful examples for dynamic energy-optimal and optimal-collection path planning
- Account for cases where the background field is affected by collection
- Account for stochasticity in ocean flow
- Compute optimal paths to clean up sargassum and harmful algae

Relevant publications: Lolla, T., Ueckermann, M. P., Yiğit, K., Haley, P. J., & Lermusiaux, P. F. J. (2012). Path planning in time dependent flow fields using level set methods. 2012 IEEE International Conference on Robotics and Automation, 166–173. https://doi.org/10.1109/ICRA.2012.6225364
Subramani, D. N., & Lermusiaux, P. F. J. (2016). Energy-optimal path planning by stochastic dynamically orthogonal level-set optimization. Ocean Modelling, 100, 57–77. https://doi.org/10.1016/j.ocemod.2016.01.006
Subramani, D. N., Lermusiaux, P. F. J., Haley, P. J., Mirabito, C., Jana, S., Kulkarni, C. S., Girard, A., Wickman, D., Edwards, J., & Smith, J. (2017). Time-optimal path planning: Real-time sea exercises. OCEANS 2017 - Aberdeen, 1–10. https://doi.org/10.1109/OCEANSE.2017.8084776

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