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Rigid Sets and Coherent Sets in Realistic Ocean Flows

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Abstract. This paper focuses on the extractions of Lagrangian Coherent Sets from realistic velocity fields obtained from ocean data and simulations, each of which can be highly resolved and non volume-preserving. We introduce two novel methods for computing two formulations of such sets. First, we propose a new "diffeomorphism-based" criterion to extract "rigid sets", defined as sets over which the flow map acts approximately as a rigid transformation. Second, we develop a matrix-free methodology that provides a simple and efficient framework to compute "coherent sets" with operator methods. Both new methods and their resulting *rigid sets* and *coherent sets* are illustrated and compared using three numerically simulated flow examples, including a high-resolution realistic, submesoscale to large-scale dynamic ocean current field in the Palau Island region of the western Pacific Ocean.

1 Introduction

The pioneering concept of Lagrangian Coherent Structures (LCS) has emerged (Haller and Yuan (2000)) to offer visualization and understanding of material transport in time-dependent fluid flows. The terminology was born from direct observations of realistic flows and refers to the persistence of distinguished material sub-domains over time (Farazmand and Haller (2012); Haller and Beron-Vera (2012); Haller (2015); Balasuriya et al. (2018); Andrade-Canto et al. (2020)). Extracting LCS is expected to allow for improved Lagrangian hazard predictions; typical ocean applications include pollution tracking (Lekien et al. (2005); Coulliette et al. (2007); Lermusiaux et al. (2019); Nolan et al. (2020)), search and rescue (Serra et al. (2020)), or ecosystem characterizations (Scales et al. (2018); Doshi et al. (2019); Paris et al. (2020)). Various useful definitions of LCS have been proposed (Farazmand and Haller (2012); Froyland et al. (2007); Haller (2015); Karrasch (2015); Onu et al. (2014); Peacock and Haller (2013); Shadden et al. (2005); Tang et al. (2010); Allshouse and Thiffeault (2012)), and there are as many computational methodologies to extract them from time-dependent (*non-autonomous*) velocity fields $\boldsymbol{v}(t,\boldsymbol{x})$. Here, the variable $\boldsymbol{x}$ denotes the spatial position over a two or three dimensional computational domain $\Omega \subset \mathbb{R}^n$ ($n = 2$ or $n = 3$). These approaches can be classified broadly into two categories (Feppon (2017)).

The first category of methods (Haller (2002); Haller et al. (2018)) focuses on the motion of individual particles whose location $\boldsymbol{x}(t)$ satisfies the Ordinary Differential Equation (ODE),

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{v}(t,\boldsymbol{x}(t)) \\ \boldsymbol{x}(0) = \boldsymbol{x}_0, \end{cases} \quad (1)$$