Persistent Lagrangian Material Coherence in Fluid and Ocean Flows Using Flow Map Composition

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Abstract

In this work, we analyze Lagrangian material coherence in dynamic open domains. We derive and apply new theory and efficient schemes to extract material sets in dynamic flow fields that remain the most or the least coherent throughout the time interval of interest, with special attention to realistic ocean domains that have multiple time-dependent inlets and/or outlets. First, the partial differential equation (PDE)-based method of composition is extended to efficiently compute flow maps in open domains, evolving a dynamic mask field without compounding numerical errors. This permits the use of existing grid based PDE solvers to compute flow maps on their dynamic non-regular domain. Inherent parallelization capabilities with accuracy as trajectory-based schemes but importantly with also an optimal grid-based resolution make this method very attractive. Second, we derive a novel approach to compute material sets in dynamic fluid flows that undergo minimal stress throughout the considered time interval. The level sets of the proposed metric, called the 'extended polar distance', yield material subdomains that remain rigid (*i.e.* only undergo translation and rotation) throughout the time interval of interest up to a certain tolerance. This metric and the corresponding persistently coherent sets and incoherent sets are computed using the PDE-based flow map computation. We further relate the extended polar distance and the diffusion barrier strength metric and show that the extended polar distance rigorously cumulates the tendency of a material subdomain to be prone to diffusion and the average strain it undergoes. We utilize the new theory and numerical methods to analyze Lagrangian coherence in analytical and realistic scenarios - an analytical unsteady double gyre flow and a realistic simulation in the Southern Pacific Ocean. The former helps us better understand the proposed theory in practice, and highlights the evolution of coherent, persistently coherent, and incoherent sets. In the latter Southern Pacific Ocean application, we find that the surface regions around Palau island are highly incoherent due to the steep topography and complex interactive dynamics. However, we also find a rigid set advected by the larger-scale currents around the Island, retrieving its shape at the end, as well as a persistently rigid set that approximately maintains its shape throughout the time interval, maximally resisting advective stretching and diffusive transport.

Keywords: Lagrangian transport, coherent structures, flow map, extended polar distance, steep topography, data assimilation

1. Introduction

Lagrangian coherent structures (LCSs) have been very helpful in improving our understanding of material transport over finite durations in unsteady fluid flows [34]. It has long been known that such coherent structures are an intrinsic property of fluid flows [69]. They often correspond to the distinguished (attracting or repelling) material surfaces or sub-domains in the fluid flow over that duration [15; 35]. There is a wide range of LCS applications, from fluid dynamic and aerodynamic analyses [65; 13; 43] and combustion studies [8; 7; 68] to biological and bio-inspired fluid flows [79; 26; 76]. However, the most prominent studies and applications of Lagrangian analyses have been in the field of geophysical and marine sciences. These include oil spill and spread predictions, pollution tracking, environmental hazards studies, and marine-biology studies [51; 72; 34; 38; 66; 39] to name a few. Several definitions of LCS exist [15; 23; 34; 45; 77] along with multiple methodologies and schemes to extract them from the given flow fields [1; 27].

So far, popular techniques for LCS studies have suggested that such relevant features can be co-dimension one (*i.e.* of dimension 1 less than that of Ω) surfaces across which the underlying flow map exhibits a