

**Stochastic Dynamically Orthogonal Modeling and  
Bayesian Learning for  
Underwater Acoustic Propagation**

by

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## Abstract

Sound waves are critical for a variety of underwater applications including communication, navigation, echo-sounding, environmental monitoring, and marine biology research. However, the incomplete knowledge of the ocean environment and acoustic parameters makes reliable acoustic predictions challenging. This is due to the sparse and heterogeneous data, as well as to the complex ocean physics and acoustics dynamics, multiscale interactions, and large dimensions. There are thus several sources of uncertainty in acoustic predictions. They include the ocean current, temperature, salinity, pressure, density, and sound speed fields, the bottom topography and seabed fields, the sources and receivers properties, and finally the model equations themselves. The goals of this thesis are to address these challenges. Specifically, we: (1) obtain, solve, and verify differential equations for efficient probabilistic underwater acoustic modeling in uncertain environments; (2) develop theory and implement algorithms for the Bayesian nonlinear inference and learning of the ocean, bathymetry, seabed, and acoustic fields and parameters using sparse data; and (3) demonstrate the new methodologies in a range of underwater acoustic applications and real sea experiments, showcasing new capabilities and leading to improved understanding.

In the first part, we derive, discretize, implement, and verify stochastic differential equations that (i) capture dominant input uncertainties in the environment (e.g., ocean, bathymetry, and seabed) and in the acoustic parameters (e.g. source location, frequency, and bandwidth), and (ii) predict the acoustic pressure fields and their probability distributions, respecting the nonlinear governing equations and non-Gaussian statistics. Starting from the acoustic Parabolic Equation (PE), we develop Dynamically Orthogonal (DO) differential equations for range-optimal acoustic uncertainty quantification. Using DO expansions for the input uncertainties, we develop the reduced-order DO-PEs theory for the Narrow-Angle PE (NAPE) and Padé Wide-Angle PE (WAPE) stochastic partial differential equations (PDEs). We verify the discretized DO-PEs in new stochastic range-independent and range-dependent test cases, and demonstrate their advantages over state-of-the-art methods for uncertainty

quantification and wave propagation in random media. Results show that a single DO-PE simulation can accurately predict stochastic range-dependent acoustic fields and their full non-Gaussian probability distributions, with computational savings of several orders of magnitude when compared to direct Monte Carlo methods.

In the second part, we extend recent nonlinear Bayesian data assimilation (DA) to the inference and learning of ocean-bathymetry-seabed-acoustic fields and parameters using sparse acoustic and oceanographic data. We combine the acoustic DO-PEs with Gaussian mixture models (GMMs) to predict probability densities in the DO subspace, allowing for efficient non-Gaussian estimation of state variables, parameters, and model functions themselves. The joint multidisciplinary estimation is enabled by state augmentation where the ocean-acoustic-bathymetry-seabed states and parameters are fit together to GMMs within the DO subspace. The new GMM-DO ocean acoustic inference system is validated by assimilating sparse data to infer the source depth, source frequency, and acoustic and environment fields and parameters in five new high-dimensional inference test cases based on state-of-the-art oceanographic and geoacoustic benchmarks. We evaluate the convergence to inference parameters and quantify the learning skill. Results show that our PDE-based Bayesian learning successfully captures non-Gaussian statistics and acoustic ambiguities. Using Bayes' law, it provides accurate probability distributions for the multivariate quantities and enables principled learning from noisy, sparse, and indirect data.

In the final part, we integrate our acoustic DO-PEs and GMM-DO frameworks with the MSEAS primitive equation ocean modeling system to enable unprecedented probabilistic forecasting and learning of ocean physics and acoustic pressure and transmission loss (TL) fields, accounting for uncertainties in the ocean, acoustics, bathymetry, and seabed fields. We demonstrate the use of this system for low to mid-frequency propagation with real ocean data assimilation in three regions. The first sea experiment takes place in the western Mediterranean Sea where we showcase the system's performance in predicting ocean and acoustic probability densities, and assimilating sparse TL and sound speed data for joint ocean physics-acoustics-source depth inversion in deep ocean conditions with steep ridges. In the second application, we simulate stochastic acoustic propagation in Massachusetts Bay around Stellwagen Bank and use our GMM-DO Bayesian inference system to assimilate TL data for acoustic and source depth inversion in shallow dynamics with strong internal waves. Finally, in the third experiment in the New York Bight, we employ our system as a novel probabilistic approach for broadband acoustic modeling and inversion. Overall, our results mark significant progress toward end-to-end ocean-acoustic systems for new ocean exploration and management, risk analysis, and advanced operations.

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