An Iterative Pressure-Correction Method for the Unsteady Incompressible Navier-Stokes Equation

by

Jean Philippe Aoussou

Submitted to the School of Engineering
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Abstract

The pressure-correction projection method for the incompressible Navier-Stokes equation is approached as a preconditioned Richardson iterative method for the pressure-Schur complement equation. Typical pressure correction methods perform only one iteration and suffer from a splitting error that results in a spurious numerical boundary layer, and a limited order of convergence in time. We investigate the benefit of performing more than one iteration.

We show that that not only performing more iterations attenuates the effects of the splitting error, but also that it can be more computationally efficient than reducing the time step, for the same level of accuracy. We also devise a stopping criterion that helps achieve a desired order of temporal convergence, and implement our method with multi-stage and multi-step time integration schemes. In order to further reduce the computational cost of our iterative method, we combine it with an Aitken acceleration scheme.

Our theoretical results are validated and illustrated by numerical test cases for the Stokes and Navier-Stokes equations, using Implicit-Explicit Backwards Difference Formula and Runge-Kutta time integration solvers. The test cases comprises a now classical manufactured solution in the projection method literature and a modified version of a more recently proposed manufactured solution.

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