High Order Stochastic Transport and Lagrangian Data Assimilation

by

Arkopal Dutt

B.Tech, Indian Institute of Technology Bombay (2015)

Submitted to the Department of Mechanical Engineering
in partial fulfillment of the requirements for the degree of

Master of Science in Mechanical Engineering

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

February 2018

Signature redacted

Author ....
Department of Mechanical Engineering
January 18, 2018

Signature redacted

Certified by..........................
Pierre F.J. Lermusiaux
Professor, Department of Mechanical Engineering
Thesis Supervisor

Signature redacted

Accepted by ....
Rohan Abeyaratne
Chairman, Department Committee on Graduate Theses

High Order Stochastic Transport and Lagrangian Data Assimilation

by

Arkopal Dutt

Submitted to the Department of Mechanical Engineering
on January 18, 2018, in partial fulfillment of the
requirements for the degree of
Master of Science in Mechanical Engineering

Abstract

Ocean currents transport a variety of natural (e.g. water masses, phytoplankton, zooplankton, sediments, etc.) and man-made materials (e.g. pollutants, floating debris, particulate matter, etc.). Understanding such uncertain Lagrangian transport is imperative for reducing environmental damage due to natural hazards and for allowing rigorous risk analysis and effective search and rescue. While secondary variables and trajectories have classically been used for the analyses of such transports, Lagrangian Coherent Structures (LCSs) provide a robust and objective description of the important material lines. To ensure accurate and useful Lagrangian hazard scenario predictions and prevention, the first goal of this thesis is to obtain accurate probabilistic prediction of the underlying stochastic velocity fields using the Dynamically Orthogonal (DO) approach. The second goal is to merge data from both Eulerian and Lagrangian observations with predictions such that the whole information content of observations is utilized.

In the first part of this thesis, we develop high-order numerical schemes for the DO equations that ensure efficiency, accuracy, stability, and consistency between the Monte Carlo (MC) and DO solutions. We discuss the numerical challenges in applying the DO equations to the unsteady stochastic Navier-Stokes equations. In order to maintain consistent evaluation of advection terms, we utilize linear centered advection schemes with fully explicit and linear Shapiro filters. We then discuss how to combine the semi-implicit projection method with new high order implicit-explicit (IMEX) linear multi-step and multistage IMEX-RK time marching schemes for the coupled DO equations to ensure further stability and accuracy. We also review efficient numerical re-orthonormalization strategies during time marching. We showcase our results with stochastic test cases of stochastic passive tracer advection in a deterministic swirl flow, stochastic flow past a cylinder, and stochastic lid-driven cavity flow. We show that our schemes improve the consistency between reconstructed DO realizations and the corresponding MC realizations, and that we achieve the expected order of accuracy.

In the second part of the work, we first undertake a study of different Lagrangian instruments and outline how the DO methodology can be applied to obtain Lagrangian variables of stochastic flow maps and LCS in uncertain flows. We then review existing methods for Bayesian Lagrangian data assimilation (DA). Disadvantages of earlier methods include the use of approximate measurement models to directly link Lagrangian variables with Eulerian variables, the challenges in respecting the Lagrangian nature of variables, and the assumptions of linearity or of Gaussian statistics during prediction or assimilation. To overcome these, we discuss how the Gaussian Mixture Model (GMM) DO Filter can be extended to fully coupled Eulerian-Lagrangian data assimilation. We define an augmented state vector of the Eulerian and Lagrangian state variables that directly exploits the full mutual information and complete the Bayesian DA in the joint Eulerian-Lagrangian stochastic subspace. Results of such coupled Eulerian-Lagrangian DA are discussed using test cases based on a double gyre flow with random frequency.

Thesis Supervisor: Pierre F.J. Lermusiaux
Title: Professor, Department of Mechanical Engineering

Acknowledgments

First and foremost, I would like to thank my advisor Prof. Pierre Lermusiaux for his wonderful guidance, incredible support and invaluable advice. I am also extremely grateful to him for being infinitely patient and understanding. I have learned a tremendous amount from him throughout the last few years both on academic and personal fronts, and hope to learn much more in the future.

I would like to thank Dr. Pat Haley for all the help with MSEAS cluster and software. I am extremely grateful to him for numerous discussions and help with my presentations. Thank you Pierre and Pat for keeping the MSEAS team in high spirits during crunch situations with your jovial comments.

We are grateful to the Office of Naval Research for support under Grants N00014-14-1-0725 (Bays-DA) and N00014-14-1-0476 (Science of Autonomy LEARNS), and to the National Science Foundation for support under grant EAR-1520825 (Hazards SEES - ALPHA), each to the Massachusetts Institute of Technology (MIT).

Thank you to Chris Mirabito for helping me with all the MSEAS website updates. Thank you to Sudip Jana for sharing Bengali jokes with me and testing the trajectory code.

The warm and "dusty" confines of the MSEAS research lab have become a second home. Thank you to Tapovan, John, Matt and Sydney for welcoming me to the lab. Jing (aka Master Shifu) has patiently answered all my questions over the years and has been a great swimming buddy along with Corbin. Thank you for sharing my enthusiasm for seafood and being such a great friend. Deepak-thank you for your life lessons, and advice on how to hone my presentation skills. It was fun navigating the AGU 2017 conference with you. Chinmay-thank you for being such a great roommate in the first year (and putting up with my weird idiosyncrasies), occasional project partner and all the reddit humor you bring into the group. Thanks to Corbin for always pushing me to go for swimming, listening to me talk on and on, and your quick wit that sparks happiness into our daily lives. Thanks to Johnathan (aka Jvo) for being the other foodie on the 8th floor of Sidpac! Florian, merci beaucoup for your insightful comments on research. Abhinav-thank you for all your help in making presentations and general camaraderie. The NSF ALPHA sea exercises in the summer of 2017 was one of the most fun experiences because of you, Chinmay, Wael, Chris and Sudip. Wael-thank you for all your general advice in life. I also would like to thank our honorary labmate Saviz.

I am also grateful to Leslie Regan, Una Sheehan, Joan Kravit and Marcia Munger for making my life less complicated by helping me in all sorts of administrative tasks.

Sidney-Pacific has truly become another home away from home. Thank you to the Heads of House - Julie, Neel, Nuria and Alberto - for all the support. Thank you to Mohammad, Jay, Raj, Elan, Olivia, Eric, Leo, Andrea, Charlie, Haozhe, Aashka, and SPEC for making life at Sidpac so awesome.

A special thank you to all my friends outside of my research lab and Sidney-Pacific. Yashovardhan Chati was one of the first to welcome me to Cambridge. Thank you to Karthik GK for laughing at all my jokes (even when they are not particularly funny), and Karthik M for being such a cool sport. Thank you to Shaswat for sharing my enthusiasm for tennis, table tennis and parathas (albeit usually frozen and bought from Shalimar). I don't know what I would have done without the "Cafe Spice Rocks" group. Thank you Ravi and Rohit for sharing my enthusiasm for Indian food (even when it is terrible). Ravi, thank you for all the memories created over table tennis, awesome lemon rice and sambar, lame jokes and cheers to many more.

Last but not least, this thesis would not be possible without the support of my family. Thank you Ashwin for always being there; in times of crisis and joy since insti. Thank you Annie, Aish, Kush and Puplu for all the WhatsApp calls and listening to my stories. No words can do justice about the amount of support that my Mamma and Baba have given me in anything that I have wanted to pursue. I also would like to thank Didi for her constant words of encouragement and motivation. This thesis is dedicated to you and the loving memory of Dada who always encouraged me to pursue my passion in science and engineering.