Neural Closure Models for Chaotic Dynamical Systems

by

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Submitted to the Department of Mechanical Engineering
in partial fulfillment of the requirements for the degree of

Master of Science in Mechanical Engineering

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

February 2023

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January 15, 2023

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Abstract

An important challenge in the problem of producing accurate forecasts of multiscale dynamics, including but not limited to weather prediction and ocean modeling, is that these dynamical systems are chaotic in nature. A hallmark of chaotic dynamical systems is that they are highly sensitive to small perturbations in the initial conditions and parameter values. As a result, even the best physics-based computational models, often derived from first principles but limited by varied sources of errors, have limited predictive capabilities for both shorter-term state forecasts and for important longer-term global characteristics of the true system. Observational data, however, provide an avenue to increase predictive capabilities by learning the physics missing from lower-fidelity computational models and reducing their various errors. Recent advances in machine learning, and specifically data-driven knowledge-based prediction, have made this a possibility but even state-of-the-art techniques in this area have not been able to produce short-term forecasts beyond a small multiple of the Lyapunov time of the system, even for simple chaotic systems such as the Lorenz 63 model. In this work, we develop a training framework to apply neural ordinary differential equation-based (nODE) closure models to correct errors in the equations of such dynamical systems. We first identify the key training parameters that have an outsize effect on the learning ability of the neural closure models. We then develop a novel learning algorithm, broadly consisting of adaptive tuning of these parameters, designing dynamic multi-loss objective functions, and an error-targeting batching process. We evaluate and showcase our methodology to the chaotic Balance Equations in an array of increasingly difficult learning settings: first, only the coefficient of one missing term in one perturbed equation; second, one entire missing term in on perturbed equation; third, two missing terms in two perturbed equations; and finally the previous but with a perturbation being two orders of magnitude larger than the state, thereby resulting in a completely different attractor. In each of these cases, our new multi-faceted training approach drastically increases both state-of-the-art state predictability (upto 15 Lyapunov times) and attractor-reproducibility. Finally, we validate our results by comparing them with the predictability limit of the chaotic

BE system under different magnitudes of perturbations.

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Acknowledgments

My first note of gratitude goes out to my advisor Prof. Pierre Lermusiaux. My time at MIT has been one of tremendous growth, both personally and professionally, thanks to your support and encouragement that has guided me throughout. Your unwavering commitment to science and research despite all odds, at times even in the face of life-altering personal adversities, inspires me and I can only hope to one day devote to my career with the same sense of duty as you do to science. I sincerely wish you the best of luck with your future endeavors.

I would also like to express my thanks to the MSEAS family both past and current. I joined the group virtually in the thick of COVID-19 which was admittedly not the best setting to socialize. But over the past 1.5 years that I have had the chance to meet with all of you, it has been a joy getting to know you better. Wael, thank you for your support, encouragement and guidance. You were my go-to person for all things research and I am really grateful for your time. Corbin, thank you for your guidance and especially the coffee walks. I really enjoyed our eclectic conversations about finance, human metabolism and much more! Anantha, it was such a pleasure having you in my office. Talking to you about anything was always so much fun and I am glad that you will not have that office all to yourself now that Yuanxhe is there. Manan, thank you so much for your advice and help, even at odd hours, in bringing this thesis to completion. Alonso, Aditya and Ellery, it has been great to hang out with you folks and see you grow as researchers. I am really excited to see all the great things you will achieve as the future of MSEAS. And finally, Tony, Pat, Abhinav, Chris, Aaron and Lisa, it has been a pleasure knowing you and working with you throughout my tenure.

My last note of gratitude goes to those who deserve it first and foremost, my family. To my parents, thank you for your unconditional love, support and patience. This degree and achievement is as much yours as it is mine and I hope I can continue to make you proud. To my brother, thank you for being my partner-in-crime at Harvard and a constant source of guidance and support throughout this time. I

always look up to you for inspiration. Thank you Aruna for holding the fort back home and keeping our minds at ease for the better part of my tenure at MIT. And last but not the least, thank you Pari for just being yourself. I hope this inspires you to do great things in life when you grow up.

I and Prof. Lermusiaux are grateful to the Office of Naval Research for partial support under grant N00014-20-1-2023 (MURI ML-SCOPE) to the Massachusetts Institute of Technology. We also thank the ML-SCOPE team for many useful discussions, especially Profs. Mickaël Chekroun and James McWilliams.