Towards Coupled Nonhydrostatic-Hydrostatic Hybridizable Discontinuous Galerkin Method

by

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Submitted to the Center for Computational Science and Engineering
in partial fulfillment of the requirements for the degree of

Master of Science in Computational Science and Engineering

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2023

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Abstract

Numerical modelling of ocean physics is essential for multiple applications such as scientific inquiry and climate change but also renewable energy, transport, autonomy, fisheries, water, harvesting, tourism, communication, conservation, planning, and security. However, the wide range of scales and interactions involved in ocean dynamics make numerical modelling challenging and expensive. Many regional ocean models resort to a hydrostatic (HS) approximation that significantly reduces the computational burden. However, a challenge is to capture and study local ocean phenomena involving complex dynamics over a broader range of scales, from regional to small scales, and resolving nonlinear internal waves, subduction, and overturning. Such dynamics require multi-resolution non-hydrostatic (NHS) ocean models. It is known that the main computational cost for NHS models arises from solving a globally coupled elliptic PDE for the NHS pressure. Optimally reducing these costs such that the NHS dynamics are resolved where needed is the motivation for this work.

We propose a new multi-dynamics model to decompose a domain into NHS and HS dynamic regions and solve the corresponding models in their subdomains, reducing the cost associated with the NHS pressure solution step. We extend a high-order NHS solver developed using the hybridizable discontinuous Galerkin (HDG) finite element methodology by taking advantage of the local and global HDG solvers for combining HS with NHS solvers. The multi-dynamics is derived, and the first version is implemented in the HDG framework to quantify computational costs and evaluate accuracy using several analyses. We first showcase results on Rayleigh Taylor instability-driven striations to evaluate computational savings and accuracy compared to the standard NHS HDG and finite-volume solvers. We highlight and discuss sensitivities and performance. Finally, we explore parameters that can be used to identify domain regions exhibiting NHS behaviour, allowing the algorithm to dynamically evolve the NHS and HS subdomains.

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Acknowledgments

I am extremely grateful to my advisor, Prof. Pierre Lermusiaux, for his mentorship and support over the last two years at MIT. I have always found your amazing work ethic and dedication to the research we do at MSEAS to be very inspiring. Thanks for giving me the opportunity to join our lab and for guiding me towards becoming a better researcher.

The daunting task of moving away from home to another country for grad school was made effortless by the incredibly welcoming people here at MSEAS. I feel so lucky to have the privilege of working in this lab and forming many great friendships. I sincerely appreciate the exceptional group dynamic at MSEAS that is very conducive to generating research ideas and this thesis would not be possible without it. The senior members of the lab, Corbin, Manan, Wael, and Aaron, you guys have been a great support system throughout and amazing company and you will be sorely missed once you graduate. A special mention to Corbin for being a great mentor and friend and for helping me through the "shallow" learning curve that comes with HDG. Anantha, Tony, Alonso and Ellery, it has been an absolute pleasure getting to know you guys and I am excited to continue working with you. Chris and Pat, you guys are so dependable and willing to help out all the time, so thanks a lot for that. Kate Nelson and Lisa Maxwell have helped me out with administrative processes time and again to ensure the smooth sailing of my program and I am indebted to them.

I would also like to thank all my friends, in and out of MIT, who have made the last couple of years in grad school so enjoyable and fun. Finally, the unwavering love, encouragement and support I receive from my family is an integral part of what keeps me going and happy so I extend my special gratitude to Amma, Appa and Kailash.

We are grateful to the Office of Naval Research for partial support under grants N00014-18-1-2781 (DRI-CALYPSO) and N00014-19-1-2693 (IN-BDA). We also thank the MIT Portugal Program for support under a MPP seed project and a Flagship program project (K2D).