

**Modeling Flow Encountering Abrupt Topography
using Hybridizable Discontinuous Galerkin
Projection Methods**

by

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B.S.E, Mechanical Engineering (Computational Mechanics)
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Submitted to the Center for Computational Engineering
in partial fulfillment of the requirements for the degree of
Master of Science in Computation for Design and Optimization
at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2017

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Abstract

In this work novel high-order hybridizable discontinuous Galerkin (HDG) projection methods are further developed for ocean dynamics and geophysical fluid predictions. We investigate the effects of the HDG stabilization parameter for both the momentum equation as well as tracer diffusion. We also make a correction to our singularity treatment algorithm for nailing down a numerically consistent and unique solution to the pressure Poisson equation with homogeneous Neumann boundary conditions everywhere along the boundary. Extensive numerical results using physically realistic ocean flows are presented to verify the HDG projection methods, including the formation of internal wave beams over a shallow but abrupt seamount, the generation of internal solitary waves from stratified oscillatory flow over steep topography, and the circulation of bottom gravity currents down a slope. Additionally, we investigate the implementation of open boundary conditions for finite element methods and present results in the context of our ocean simulations. Through this work we present the hybridizable discontinuous Galerkin projection methods as a viable and competitive alternative for large-scale, realistic ocean modeling.

Thesis Supervisor: Pierre F.J. Lermusiaux

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Acknowledgments

I want to begin by thanking my advisor, Prof. Pierre Lermusiaux. I believe the role of an advisor is not to answer all of your questions but to question all of your answers. During my time working with Pierre, he has instilled in me a drive for excellence and attention to detail. In this regard, Pierre has always pushed me to rigorously defend my claims and helped me develop the confidence and resolve to succeed as a researcher. I am very grateful to be part of the MSEAS family and look forward to working with MSEAS in the future.

I also want to thank the research scientists and postdoctoral researchers of MSEAS. It has been an absolute pleasure working with you all. Thank you so much for sharing your wisdom and seemingly unlimited knowledge of all things numerics, geophysical fluid dynamics, and computing. Of course, I must specially thank Dr. Chris Mirabito for the many long hours discussing the theory and implementation of HDG. I appreciate your patience and willingness to answer all my questions as I worked through the intricacies of the HDG code.

The MSEAS family would not be complete without my labmates. You guys are the best. Thank you all for making my time at MIT so great. Furthermore, I would like to express my gratitude towards Matt Ueckermann for laying the foundation for this work. He is the original architect for the MSEAS HDG code upon which much of this work is built.

Attending MIT would not have been possible for me without the support of Sandia National Laboratories through the Critical Skills Masters Degree program. We also thank the National Oceanographic Partnership Program (NOPP) under grant N00014-15-1-2597 (Seamless Multiscale Forecasting) and the Office of Naval Research under grant N00014-15-1-2626 (FLEAT) for partial research support to C. Mirabito, P. Haley, and P. Lermusiaux.

Lastly, I thank my parents and my brother whose love and support helped me get to where I am today.