Application of the MITgcm Modeling Framework for Global Ocean State Estimation in ECCO

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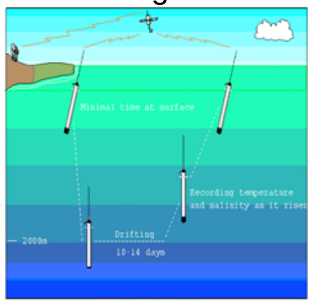
http://www.ecco-group.org

Outline

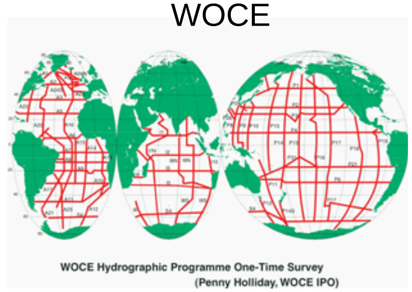
- The ocean state estimation framework
  - The optimization / optimal control problem
  - The MIT general circulation model (MITgcm) and its adjoint
  - Observations, uncertainties, control variables
  - Examples of some residual misfits
- Some example results
  - Decadal variations in North Atlantic heat and volume transport
  - Decadal variations in global sea-level patterns
- Future directions
  - Coupled ocean/sea-ice estimation
  - A truly global grid including the Arctic
  - Moving toward higher resolution
- OpenAD: a new tool for automatic differentiation
  - MITgcm sensitivity application
- Outlook
  - Long-term goals
  - Problems and challenges

Ocean State Estimation

Argo

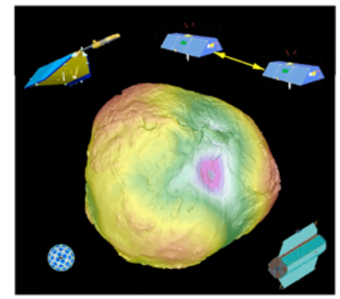


WOCE

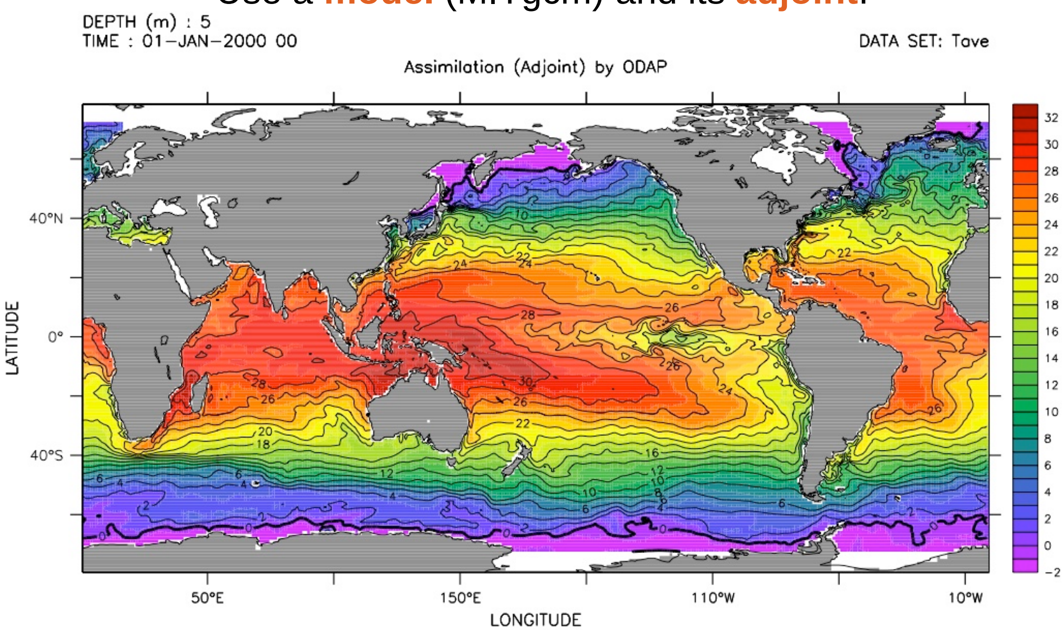


T/P, Jason

GRACE



How to synthesize? Estimation/optimal control problem:
Use a **model** (MITgcm) and its **adjoint**:



Temperature (Deg C)

Least-squares optimization / optimal control problem

- **Given:**
  - a set of (possibly different types of) observations
  - a numerical model & set of initial/boundary conditions, parameters
- **Question:** (estimation / optimal control problem)
  Find "*optimal*" model trajectory consistent with available observations
- **Approach:** seek minimum of least square cost function

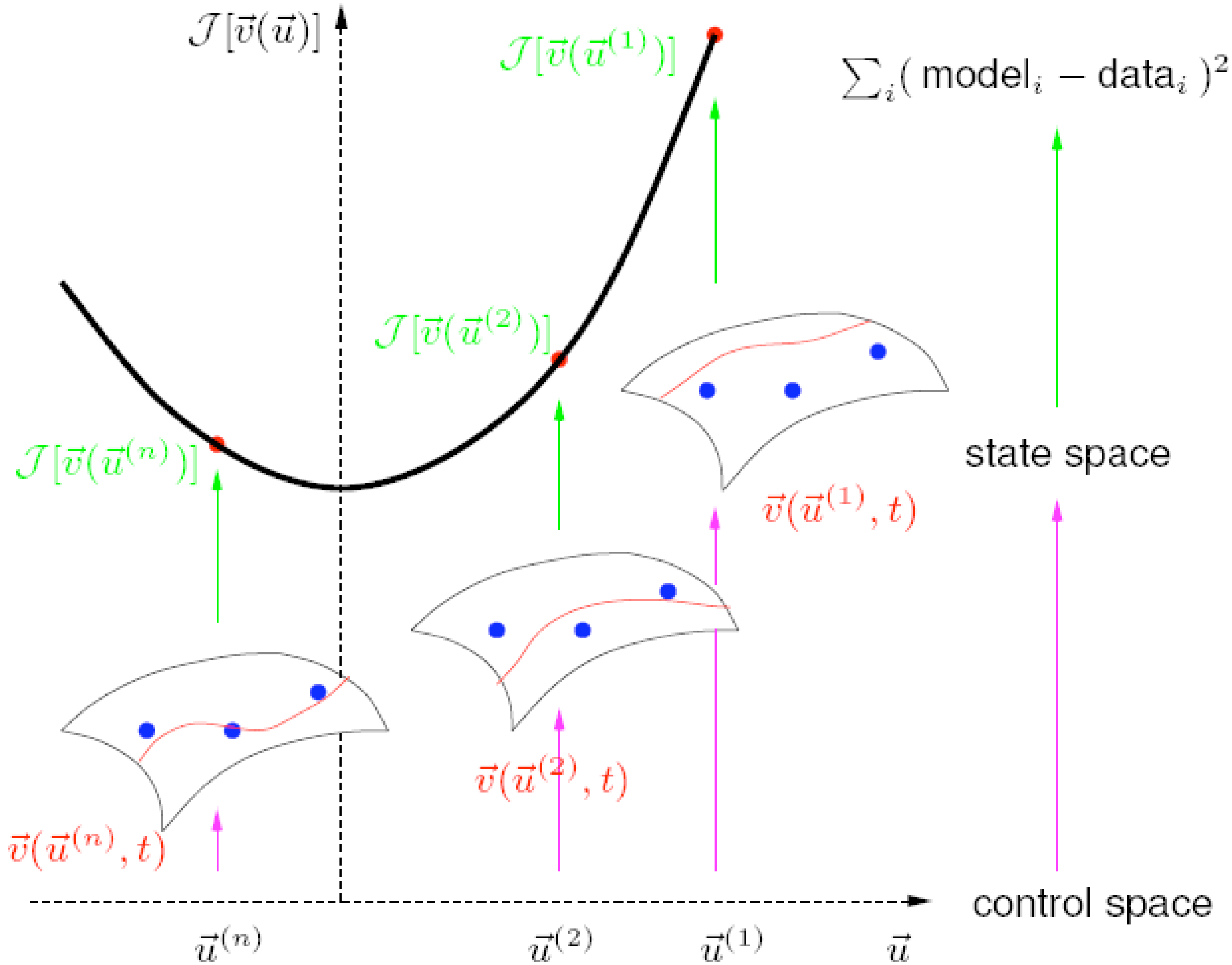
$$\min_{\vec{u}} \{\mathcal{J}(\vec{u})\} = \min_{\vec{u}} \left\{ \sum_i [\,\text{model}_i(\vec{u}) - \text{data}_i\,]^2 \right\}$$

$\longrightarrow$ seek $\boxed{\vec{\nabla}_u \mathcal{J}(\vec{u})}$ to infer update $\Delta\vec{u}$ from variation of controls $\vec{u}$

$$\vec{u}^{n+1} = \vec{u}^{n} + \Delta\vec{u}$$

- **Results:** see ECCO (Stammer et al., 2002/03)
  - optimal/consistent ocean state estimate
  - adjusted initial/boundary value estimates

Iterative optimization via gradient descent



Some algebra

Need $\vec{\nabla}_u \mathcal{J}|_{u_0}$ of $\mathcal{J}(\vec{u}_0) \in I\!R^1$ w.r.t. control variable $\vec{u} \in I\!R^m$

$\mathcal{J}$:	$\vec{u}$	$\mapsto$	$\vec{v} = \mathcal{M}(\vec{u})$	$\mapsto$	$\mathcal{J}(\mathcal{M}(\vec{u}))$
TLM :	$\delta\vec{u}$	$\mapsto$	$\delta\vec{v} = M \cdot \delta\vec{u}$	$\mapsto$	$\delta\mathcal{J} = \vec{\nabla}_u \mathcal{J} \cdot \delta\vec{u}$
ADM :	$\delta^*\vec{u} = \vec{\nabla}_u \mathcal{J}^T$	$\leftarrow\!\!\shortmid$	$\delta^*\vec{v}$	$\leftarrow\!\!\shortmid$	$\delta\mathcal{J}$

- $\vec{v} = \mathcal{M}(\vec{u})$ nonlinear model
- M , M^T tangent linear (TLM) / adjoint (ADM)
- $\delta\vec{u}$, $\delta^*\vec{u}$ perturbation / dual (or sensitivity)

$$\boxed{\vec{\nabla}_u \mathcal{J}^T|_{\vec{u}} \quad = \quad M^T|_{\vec{v}} \cdot \vec{\nabla}_v \mathcal{J}|_{\vec{v}}}$$

TLM : $m\,(\sim n_x n_y n_z)$ integrations @ $1 \cdot$ (#forward)

ADM : 1 integration @ $\gamma \cdot$ (#forward)

Automatic Differentiation (AD)

- Model code

$$\vec{v} = \mathcal{M}_\Lambda \left(\mathcal{M}_{\Lambda-1} \left(\ldots \ldots \left(\mathcal{M}_0 \left(\vec{u} \right) \right) \right) \right)$$

- Adjoint code

$$\delta^* \vec{u} = M_0^T \cdot M_1^T \cdot \ldots \ldots \cdot M_\Lambda^T \cdot \delta^* \vec{v}$$

- Automatic differentiation:

> each line of code is elementary operator $\mathcal{M}_\lambda$
> $\longrightarrow$ rules for differentiating elementary operations
> $\longrightarrow$ yield elementary Jacobians M_λ
> $\longrightarrow$ composition of M_λ's according to chain rule
> yield full tangent linear / adjoint model

- TAMC / TAF source-to-source tool (Giering & Kaminski, 1998)

$$\left. \begin{array}{l} \bullet \text{ model } \mathcal{M} \\ \bullet \text{ independent } \vec{u} \\ \bullet \text{ dependent } \mathcal{J} \end{array} \right\} \xrightarrow{\text{TAMC / TAF}} \left\{ \begin{array}{l} \text{TLM } M, \text{ or} \\ \text{ADM } M^T, \text{ or} \\ \text{gradient } \delta^* \vec{u} = \vec{\nabla}_u \mathcal{J} \end{array} \right.$$

TAF: Transformation of Algorithms in Fortran

http://www.fastopt.de

- Commercial successor of TAMC
- Source-to-source tool for F77/F90/F95 code
- Produces readable derivative code
- Recompute all by default + Efficient Recomputation Algorithm (ERA)
- Flow directives enable
  - insertion of taping or checkpointing directives to disk or memory
  - ignore passive routines
  - active I/O handling
  - self-adjointness
  - application of implicit function theorem for iterative loops
  - handle hand-written derivative code
  - adjoint checkpointing (“Divided Adjoint” - DIVA)

Has been applied to various large-scale geophysical (Earth system) and high-performance CFD codes

Giering & Kaminski: Recipes of adjoint code construction. ACM Trans. Math. Software (TOMS), 1998.

Storing vs. recomputation: the critical feature of checkpointing for time-stepping algorithms (I)

Example of a simple time-stepping box model

```
DO t = 1, nTimeSteps
```

- calculate density

$$\rho = -\alpha T + \beta S$$

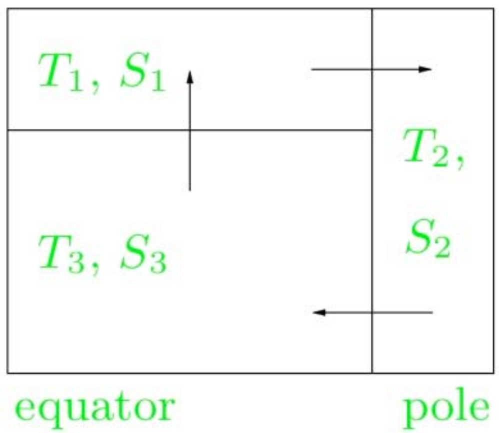
- calculate thermohaline transport

$$U = U(\rho(T, S))$$

- calculate tracer advection

$$\frac{d}{dt}Tr = f(Tr, U)$$

calculate timestepping and update tracer fi elds $Tr = \{T, S\}$




```
END DO
```

Storing vs. recomputation: the critical feature of checkpointing for time-stepping algorithms (II)

$$\begin{aligned} \frac{dT_3}{dt} &= U(T_3 - T_2), \quad \text{for } U \geq 0 \\ \mathtt{diffT3} &= \mathtt{u} * (\mathtt{T3} - \mathtt{T2}) \end{aligned}$$

derivative $\delta\mathtt{diffT3}$:

$$\delta\mathtt{diffT3} = \frac{\partial\mathtt{diffT3}}{\partial\mathtt{U}}\delta\mathtt{U} + \frac{\partial\mathtt{diffT3}}{\partial\mathtt{T}_2}\delta\mathtt{T}_2 + \frac{\partial\mathtt{diffT3}}{\partial\mathtt{T}_3}\delta\mathtt{T}_3$$

in matrix form:

$$\begin{pmatrix} \delta\mathtt{diffT3} \\ \delta\mathtt{T}_3 \\ \delta\mathtt{T}_2 \\ \delta\mathtt{U} \end{pmatrix}^{\lambda} = \begin{pmatrix} 0 & -\mathtt{U} & \mathtt{U} & \mathtt{T3} - \mathtt{T1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \delta\mathtt{diffT3} \\ \delta\mathtt{T}_3 \\ \delta\mathtt{T}_2 \\ \delta\mathtt{U} \end{pmatrix}^{\lambda-1}$$

Storing vs. recomputation: the critical feature of checkpointing for time-stepping algorithms (III)

Transposed relationship yields:

$$\begin{pmatrix} \delta^* \mathtt{diffT3} \\ \delta^* \mathtt{T}_3 \\ \delta^* \mathtt{T}_2 \\ \delta^* \mathtt{U} \end{pmatrix}^{\lambda-1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -\mathtt{U} & 1 & 0 & 0 \\ \mathtt{U} & 0 & 1 & 0 \\ \mathtt{T3}-\mathtt{T1} & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \delta^* \mathtt{diffT3} \\ \delta^* \mathtt{T}_3 \\ \delta^* \mathtt{T}_2 \\ \delta^* \mathtt{U} \end{pmatrix}^{\lambda}$$

and thus adjoint code:

```
adT3      = adT3        - u*addiffT3
adT2      = adT2        + u*addiffT3
adU       = adu   + (T3-T2)*addiffT3
addiffT3 = 0
```

Note: state T2, T3, U are required to evaluate derivative at each time step, in reverse order!

$\longrightarrow$ *TANGENT* linearity

Storing vs. recomputation: the critical feature of checkpointing for time-stepping algorithms (IV)

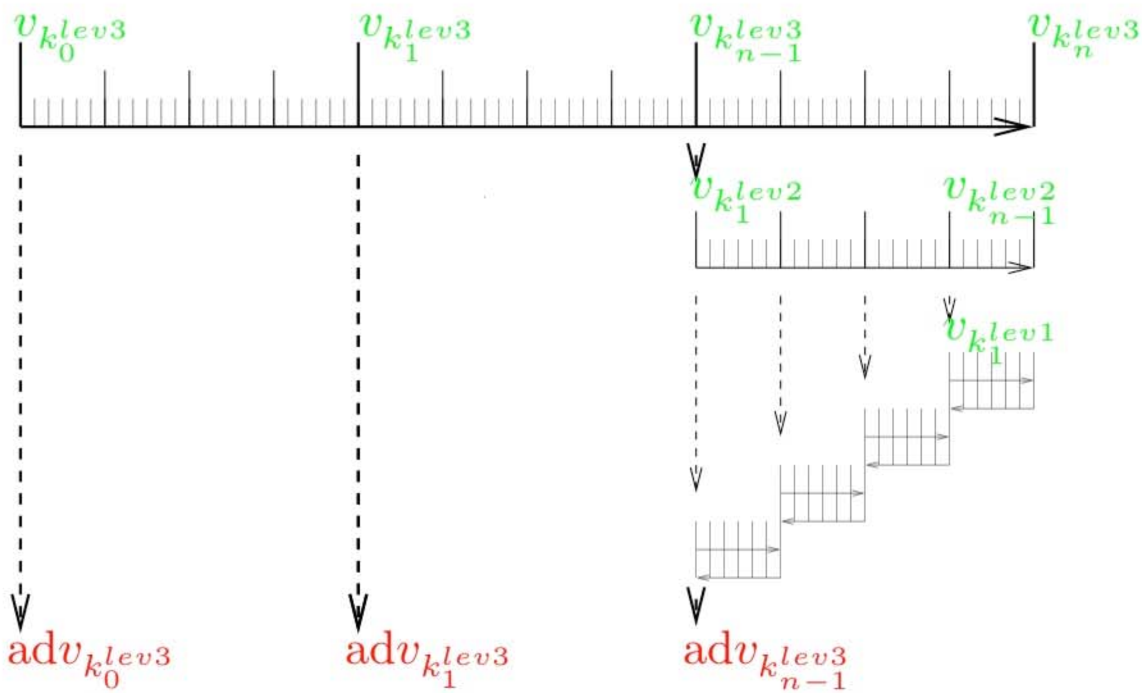
- *Adjoint = transpose of TLM*

→ evaluated in reverse order

→ model state at every time step required in reverse

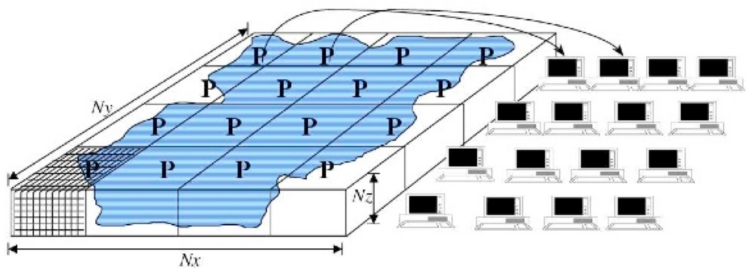
→ all state stored or recomputed

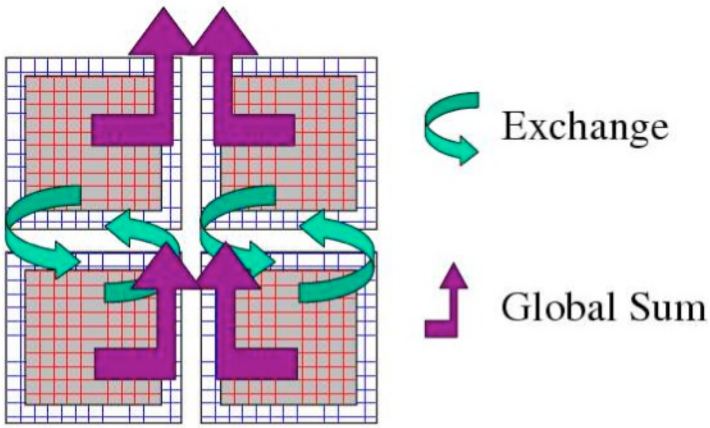
- *Solution: Checkpointing*
(e.g. Griewank, 1992, Restrepo et al., 1998) balances storing vs. recomputation



Adjoint parallel and I/O primitives

- domain decomposition (tiles) & overlaps (halos)
- split into extensive on-processor and global phase





Global communication/arithmetic op.'s supported by MITgcm's intermediate layer (WRAPPER) which need hand-written adjoint forms

	operation/primitive	forward		reverse
•	communication (MPI,...):	send	$\longleftrightarrow$	receive
•	arithmetic (global sum,...):	gather	$\longleftrightarrow$	scatter
•	active parallel I/O:	read	$\longleftrightarrow$	write

New controls, parameter activity, and *incremental* improvement of an AD-generated adjoint model

- **Total depth in partial cell formulation**

$$H(i,j) = \sum_k h_c(i,j,k)\Delta r_f(k)$$

  - $\Delta_r(k)$: full cell thickness of layer k
  - $h_c(i,j,k)$: fractional cell thickness

$$h_c(i,j,k) \begin{cases} = 1 & \text{for} \quad k < k_{low} \\ = h_{low}(i,j) \in (0,1] & \text{for} \quad k = k_{low} \\ = 0 & \text{for} \quad k > k_{low} \end{cases}$$

- **Variation of H**

$$\delta H(i,j) = \sum_k \delta h_c(i,j,k)\,\Delta r_f(k) = \delta h_{low}(i,j)\,\Delta r_f(k_{low})$$

- **Zonal volume transport U through cell face A_x**

$$U = uA_x = u\Delta x\,\Delta r_f\,h_w$$

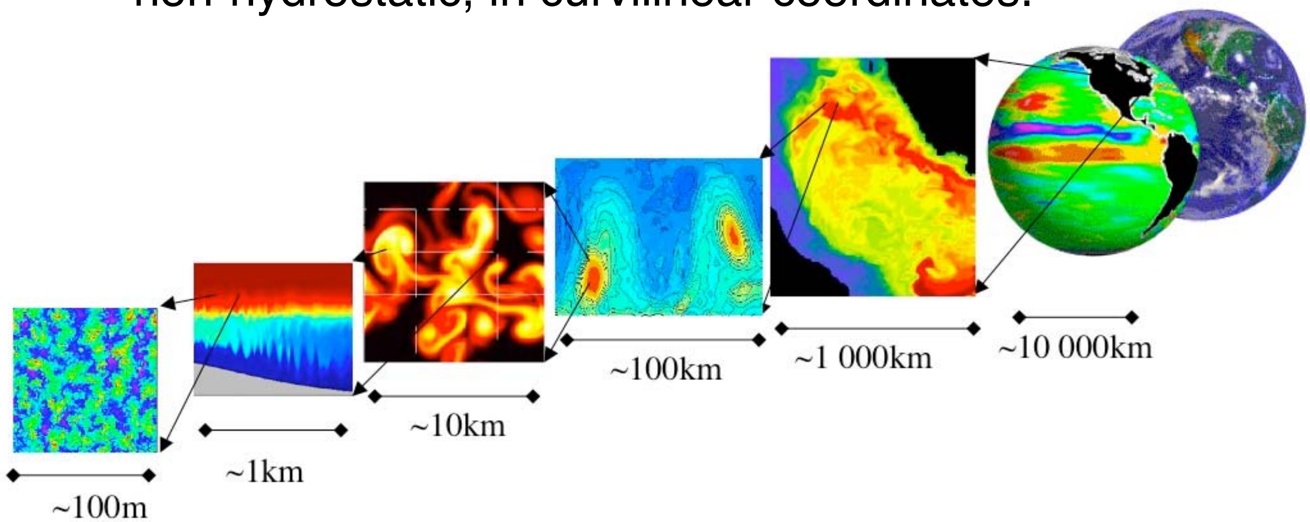
- **δU is quadratic for active A_x: product rule**

$$\delta U = \delta u\,A_x + u\,\delta A_x = \Delta x\,\Delta r_f\;(\delta u\,h_w + u\,\delta h_w)$$

$\longrightarrow$ A modified/extended adjoint code is required!

The MIT general circulation model (MITgcm)

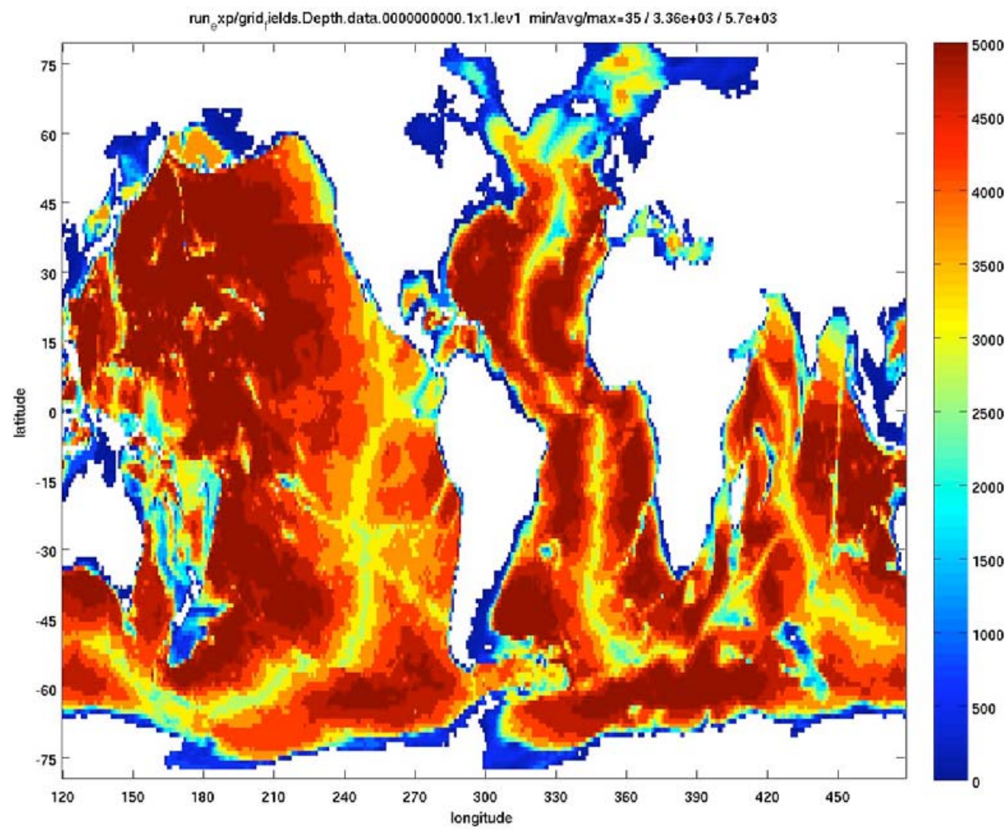
Parallel implementation of a general-purpose grid-point algorithm for a Boussinesq or non-Boussinesq fluid, hydrostatic or non-hydrostatic, in curvilinear coordinates.



- z-level or pressure vertical coordinates (ocean - atmosphere isomorphism)
- nonlinear free surface and z^* vertical coordinates
- finite-volume formulation with partial cells
- various parameterization schemes (GM/Redi, KPP, Leith, Smagorinsky)
- thermodynamic/dynamic sea-ice model (Hibler-type)
- ocean biogeochemical model
- cubed-sphere global grid topology

The ECCO-GODAE setup

- 1 degree horiz. Resolution, covering 80N to 80S
- 23 vertical levels
- GM/Redi eddy parameterization, KPP vertical mixing scheme
- covers 1992 to 2004 (nov through 2006)
- forcing: 6-hourly NCEP air-sea fluxes



Control variables

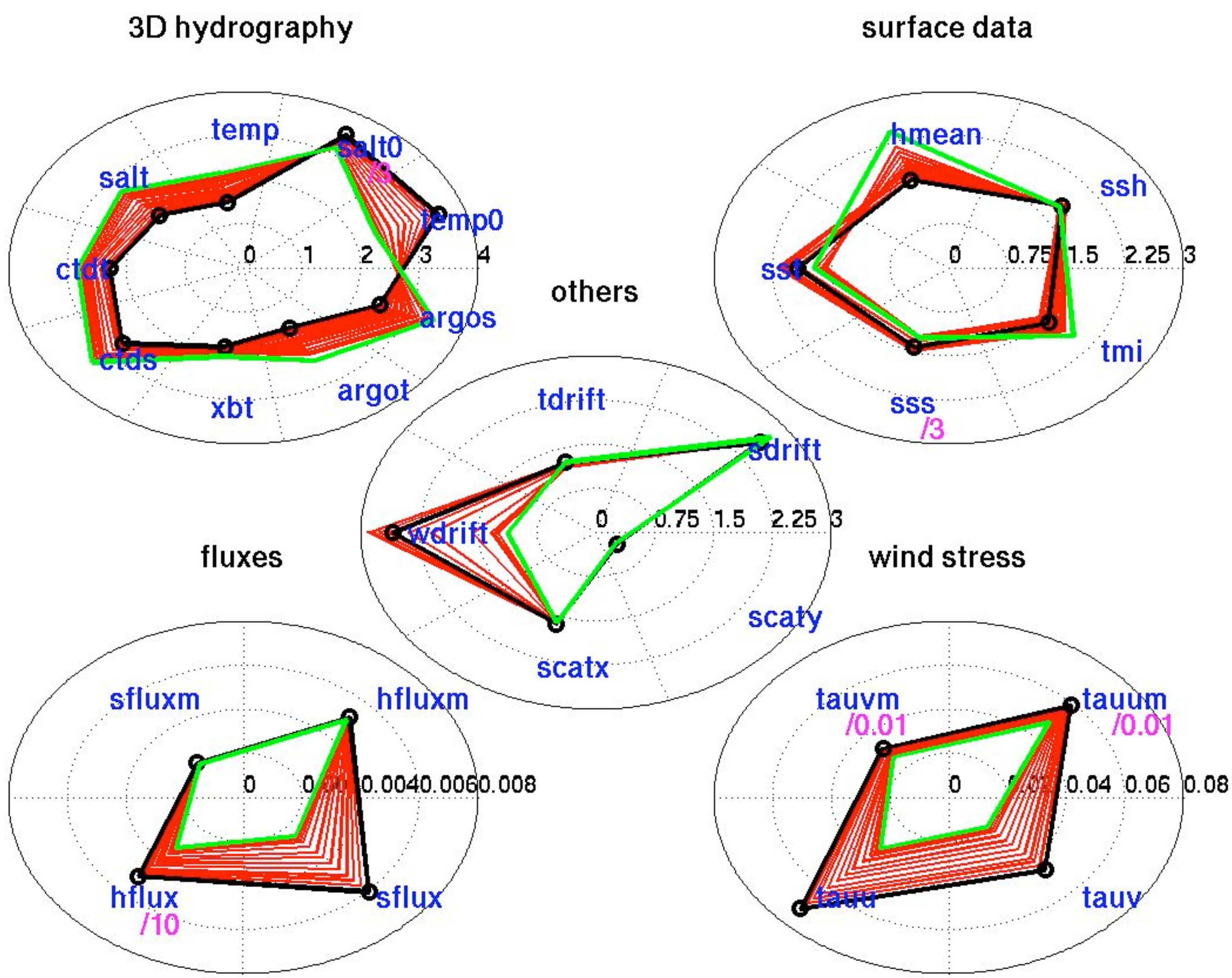
- 3-dim. initial conditions
  - temperature, salinity
- 2-dim. time-varying surface forcings:
  - Version 2:
    - heat flux, freshwater flux,
    - zonal/meridional windstress
  - Version 3:
    - surface air temperature, specific humidity, precipitation,
    - downwelling shortwave radiation,
    - zonal/meridional wind speed
- 3-dim. internal model parameters (experimental)
  - mixing coefficients (Stammer, 2005)
  - eddy stress parameterization (Ferreira et al., 2005)
  - bottom topography (Losch and Heimbach, 2007)

A large-scale optimal control problem

ECCO-GODAE state estimation: problem size (version 2)

- Dimensionality:
  - grid @ $1^\circ \times 1^\circ$ resolution: $n_x \cdot n_y \cdot n_z$ = 360 · 160 · 23 — $1,324,800$
  - model state: 17 3D + 2 2D fields — $\sim 2 \cdot 10^7$
  - timesteps: 15 years @ 1-hour time step — $131,400$
  - control vector — $\sim 1.5 \cdot 10^8$
    - initial temperature (T), salinity (S)
    - time-dependent surface forcing (every 2 days)
  - cost function: observational elements: — $\sim 2.5 \cdot 10^8$
- Computational size (per iteration):
  - 96 processors @ 2GB per proc. (SGI Altix)
  - I/O: 20 GB input, 35GB output
  - time: 28 hours per iteration @ 96 processors
- What we would ideally want:
  - $1/16^\circ \times 1/16^\circ$ resol., 1000 years, full model error covariance ...

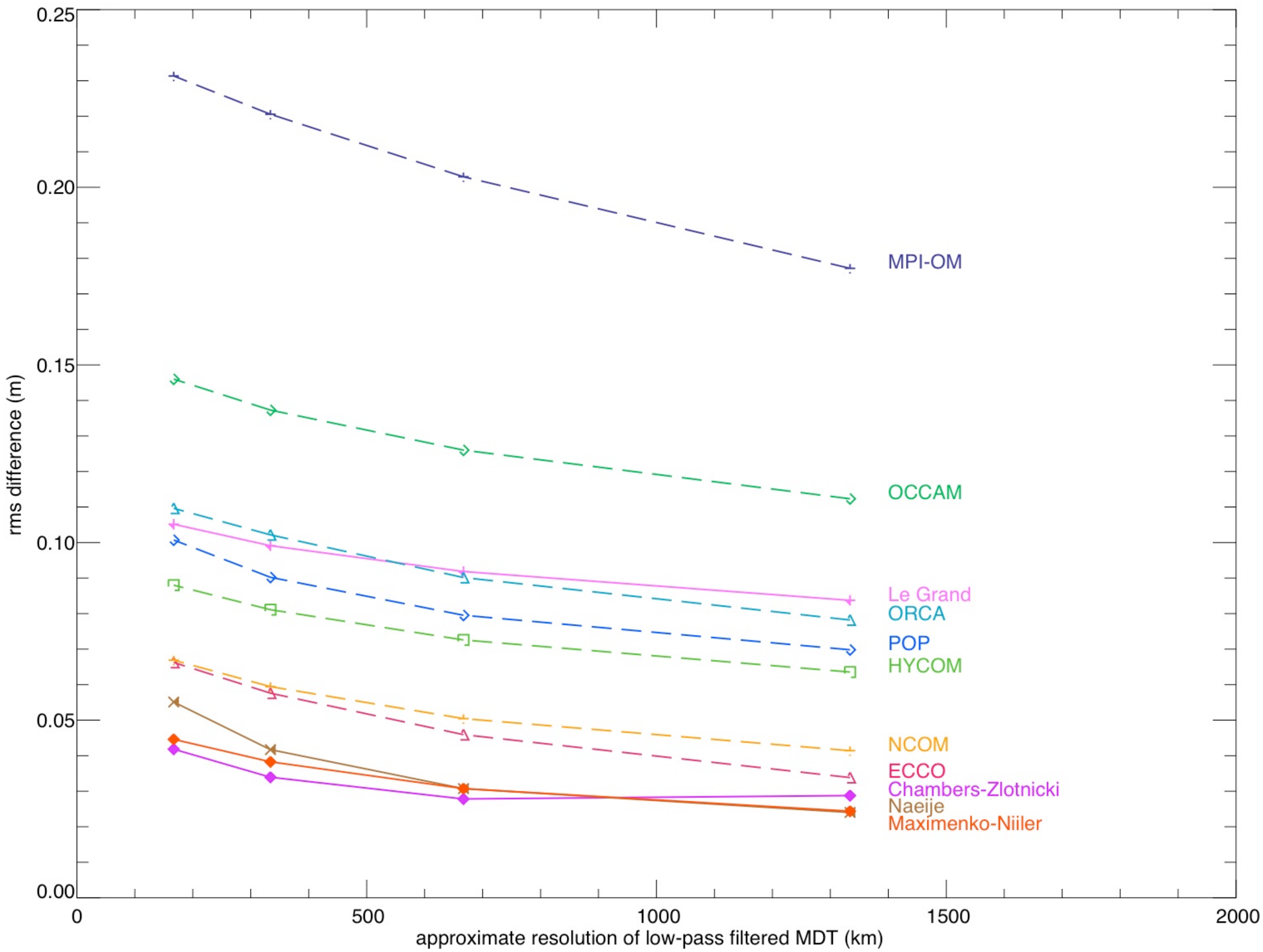
Misfits: summary of cost function reduction iteration 193 vs. 177 (G. Forget)



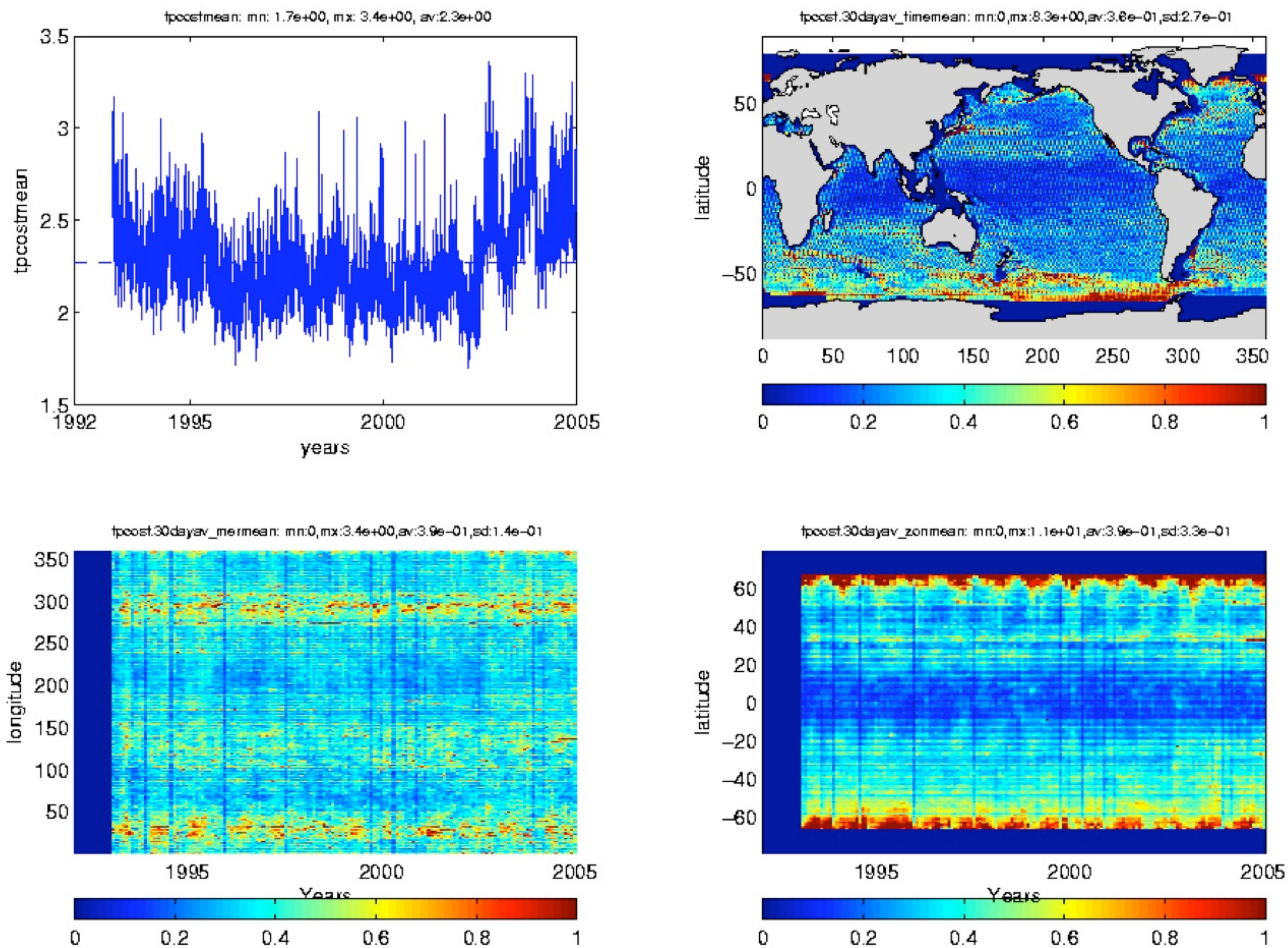
Misfits: Mean Dynamic Topography (MDT) from satellite gravity

RMS difference: GRACE-based MDT by CLS (Rio, 1995) vs. various OSE products

Vossepoel, JGR (2007)



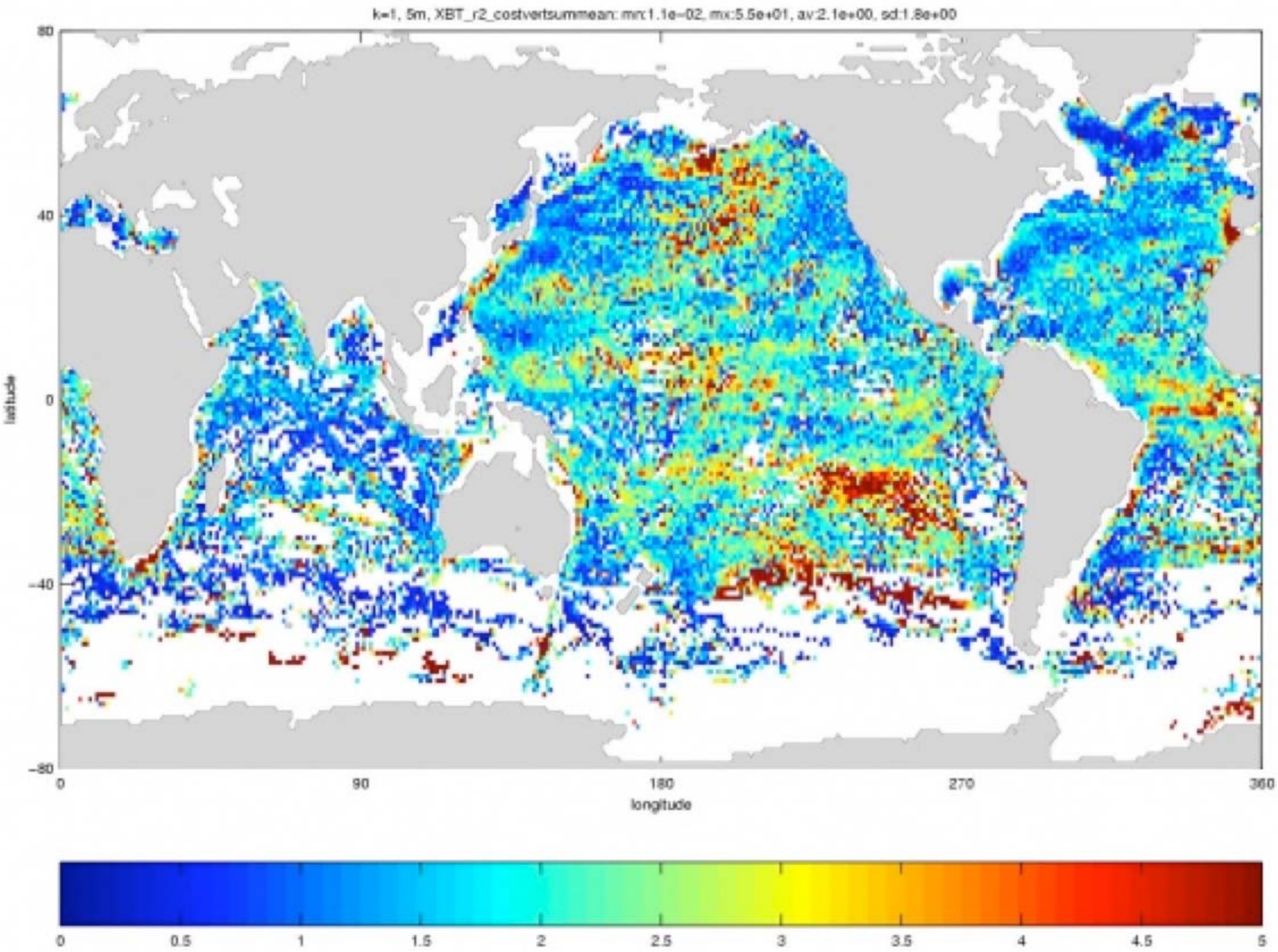
Misfits: Sea Level Anomaly (SLA) from satellite altimetry



SLA ***cost*** of model vs. T/P, Jason for ECCO-GODAE v2.177

cost *= (model - obs)^2 / (prior uncertainty)^2*

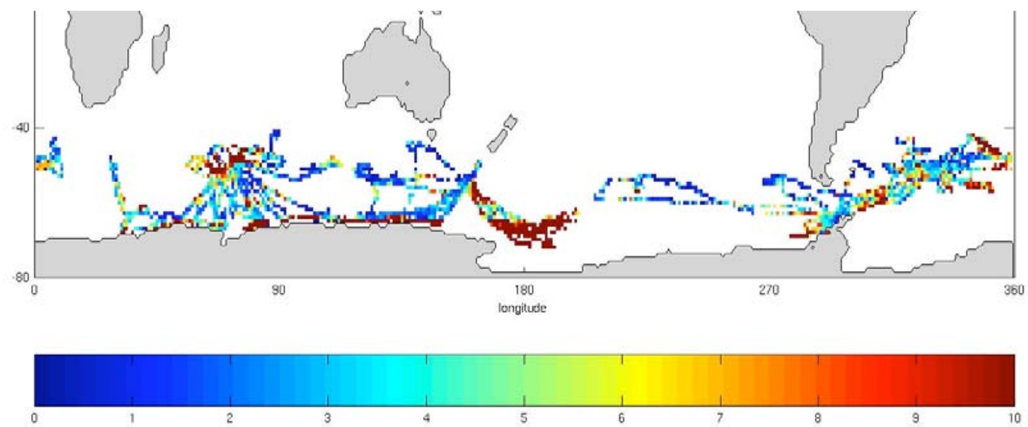
Misfits: in-situ XBT costs for v2.193



Southern Elephant Seals as Oceanographic Samples (SEaOS)

- CTD-type observations from seals in SO

Sea Mammal Research Unit,
University St. Andrews, UK,
British Antarctic Survey
(M. Meredith)



2006/11/8 11:44:13 ps: SEAOS_iter3.05_T_wtedav_cost_vertsummean_1dep__10.psc, data: /sea/raid0/diana/seaos/T_wtedav_cost_vertsummean

Application: Decadal variations in Atlantic poleward heat and mass transports (I)

Vol 438|1 December 2005|doi:10.1038/nature04385 nature

LETTERS

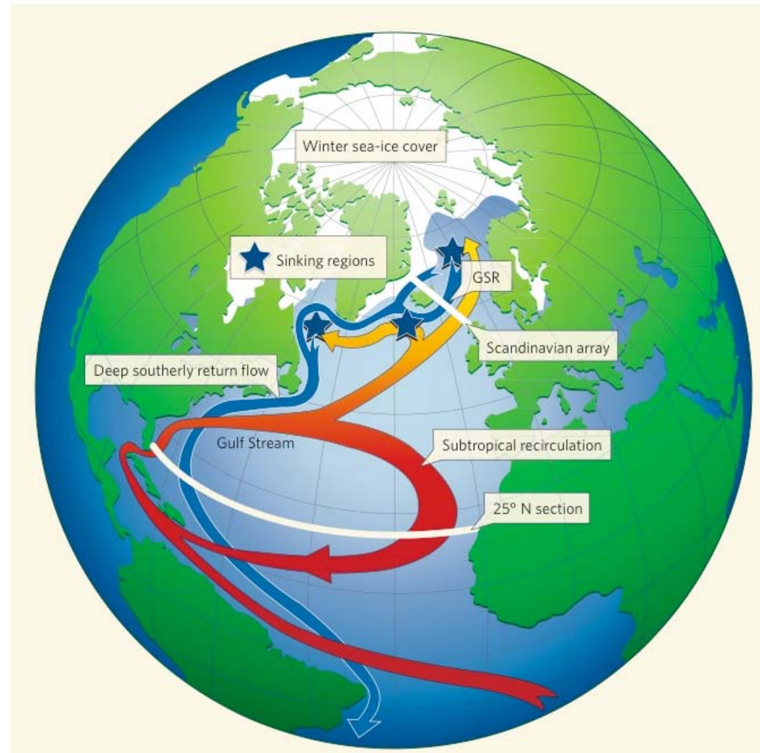
Slowing of the Atlantic meridional overturning circulation at 25° N

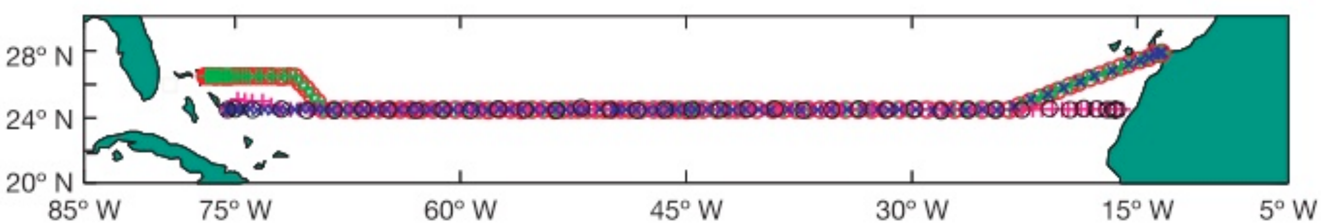
Harry L. Bryden[1], Hannah R. Longworth[1] & Stuart A. Cunningham[1]

Table 1 | Meridional transport in depth classes across 25° N

	1957	1981	1992	1998	2004
Shallower than 1,000 m depth					
Gulf Stream and Ekman	+35.6	+35.6	+35.6	+37.6	+37.6
Mid-ocean geostrophic	−12.7	−16.9	−16.2	−21.5	−22.8
Total shallower than 1,000 m	+22.9	+18.7	+19.4	+16.1	+14.8
1,000–3,000 m	−10.5	−9.0	−10.2	−12.2	−10.4
3,000–5,000 m	−14.8	−11.8	−10.4	−6.1	−6.9
Deeper than 5,000 m	+2.4	+2.1	+1.2	+2.2	+2.5

Values of meridional transport are given in Sverdrups. Positive transports are northward.

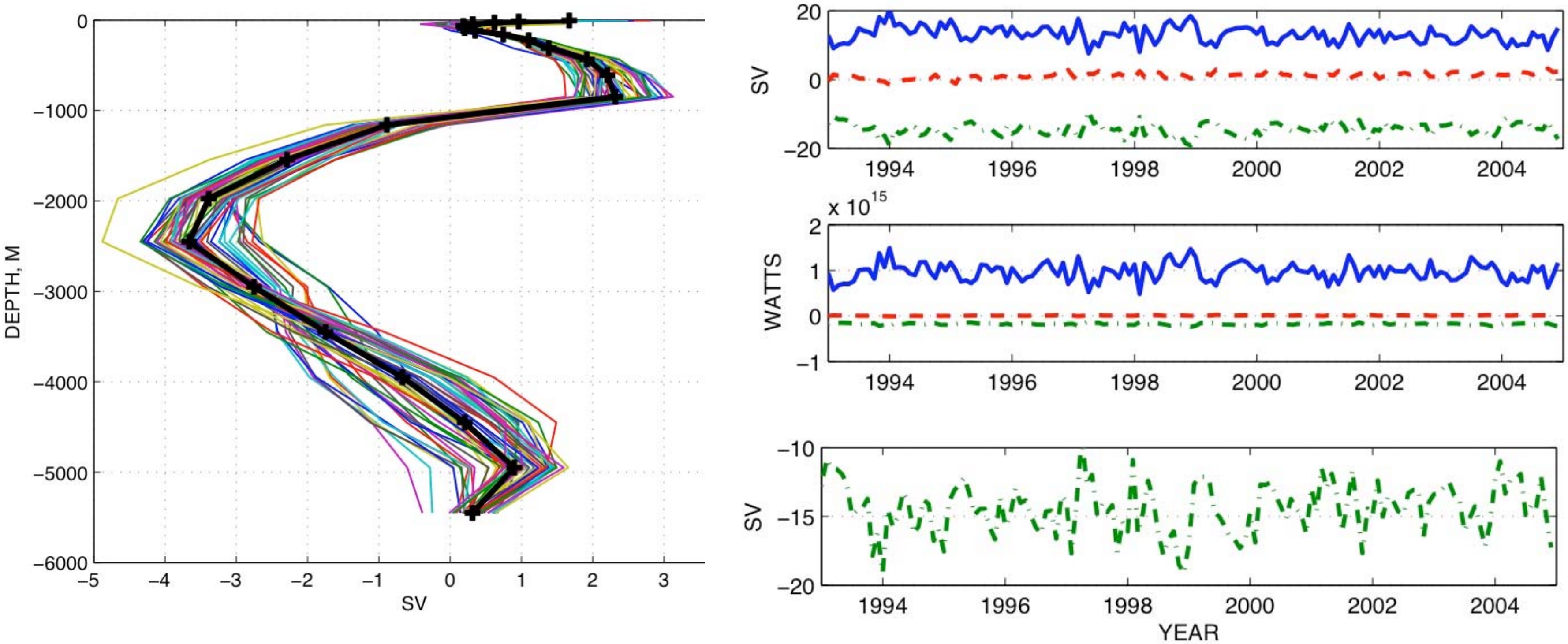




Application: Decadal variations in Atlantic poleward heat and mass transports (II)

- Historical hydrographic section A5 at 26^{o}N the North Atlantic
  - Bryden et al 2005 (Nature): “Slowing of the Atlantic overturning circulation”
  - UK RAPID program to measure elements of MOC via moorings

- The ECCO-GODAE data-constrained estimate for 1993-2004

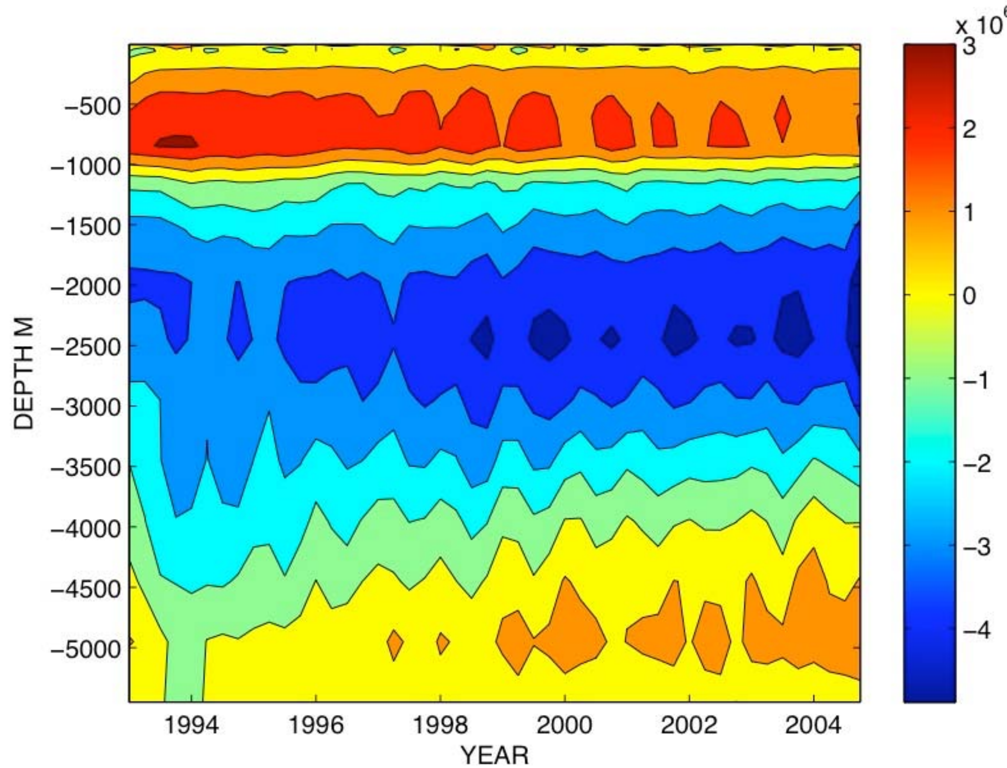


Application: Decadal variations in Atlantic poleward heat and mass transports (III)

Transports & Trends:

Heat transport	
mean	(0.84 ± 0.18) PW
trend	(-1.1 ± 4,3)$\times 10^{-3}$ PW/yr
	(not significant)

Volume transport	
mean	13.2 ± 1.8 Sv
trend	-0.19 ± 0.05 Sv/yr
	(marginally significant)



Wunsch and Heimbach, 2006 (J. Phys. Oceanogr.)

- ECCO-GODAE estimate yields no significant trend in heat transport, and only marginally significant trend in volume transport;
- serious sampling/aliasing issues expected in the Bryden et al. estimate;
- results remain fragile in view of remaining uncertainties

Application: decadal sea-level patterns (I)

ECCO-GODAE *solution version 2, iteration 216 (v2.216)*

- ***Science goal:***

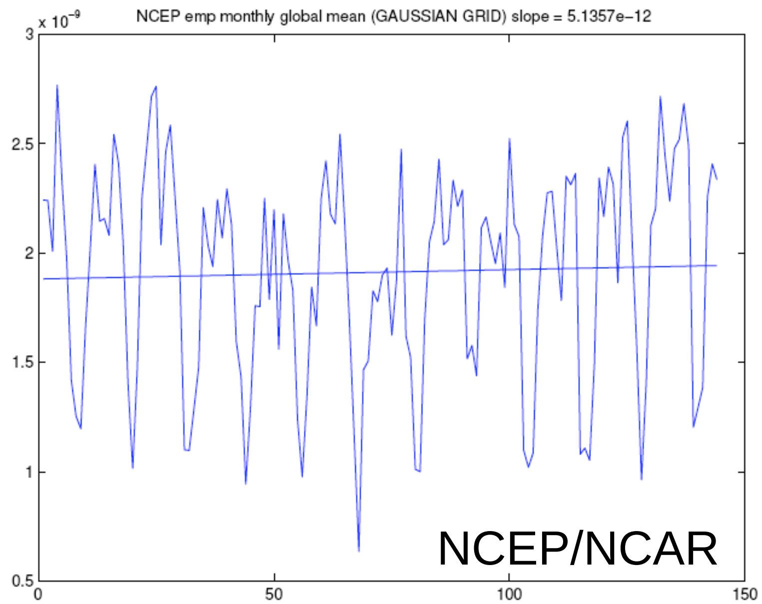
  investigate patterns of decadal sea-level variability and its partition into steric and mass-change contributions
- ***What is needed:***
  - accurate heat and freshwater forcing
  - accurate treatment of surface boundary condition
  - numerical accuracy, ensuring tracer conservation
  - closed property budgets
  - uncertainty estimates in altimetric data

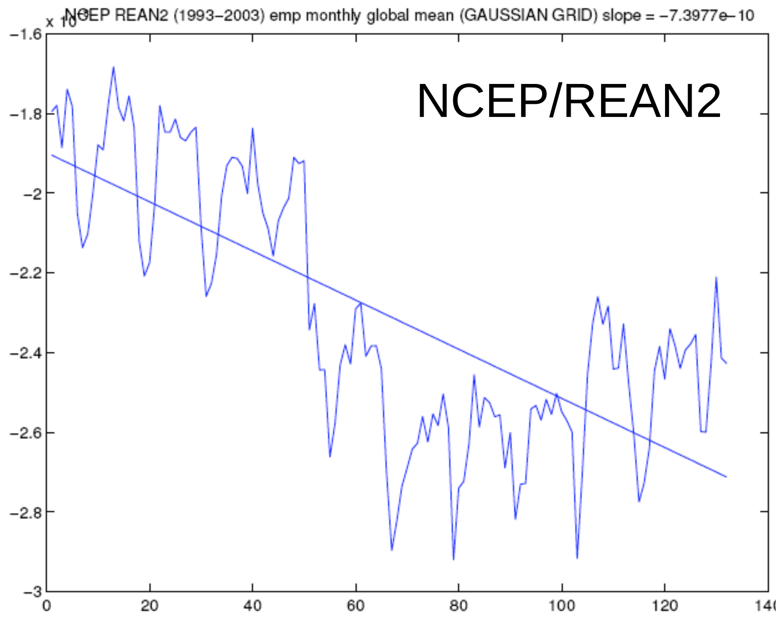
→ ***Accuracy required*** to match 2-3 mm/y (sub-)global mean sea-level rise as inferred from altimetry

Application: decadal sea-level patterns (II)

- Need to remove air-sea flux imbalances

	mean [cm/year]	intercept [mm/sec]	slope [mm/sec^2]
NCEP/NCAR-I ocean $E-P$	15.1	4.90E-9	9.29E-12
NCEP/NCAR-I ocean $E-P-R$	6.2	1.92E-9	9.29E-12
ECCO-GODAE ocean $E-P-R$	3.9	1.35E-9	-14.99E-12
NCEP/NCAR-I global $E-P$	6.1	$\sim$ 1.90E-9	5.14E-12
NCEP/DOE-II global $E-P$	-73.9	$\sim$ -19.00E-9	-740.00E-12



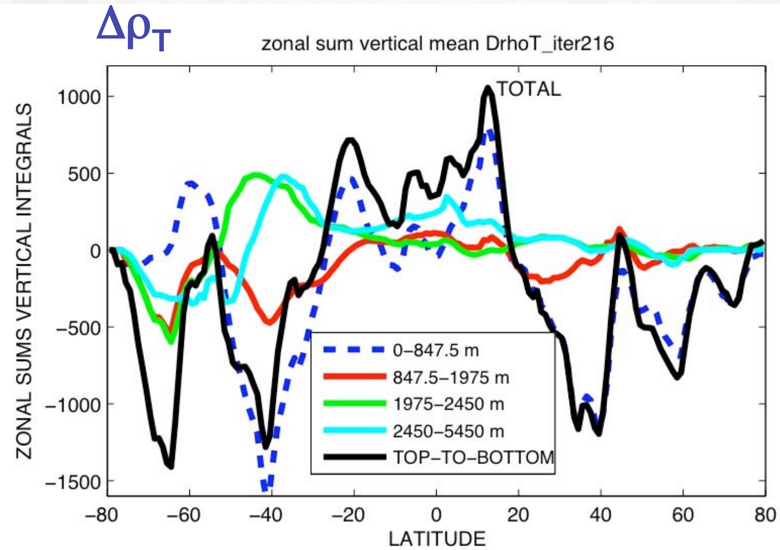


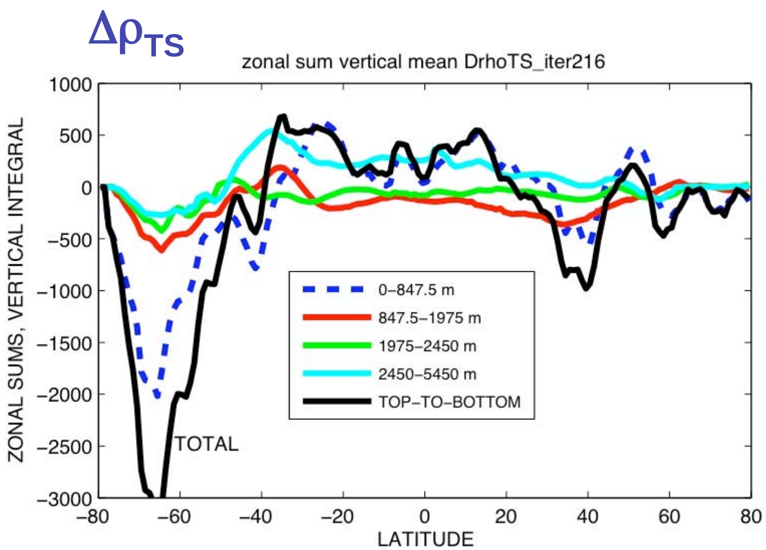
Application: decadal sea-level patterns (III)

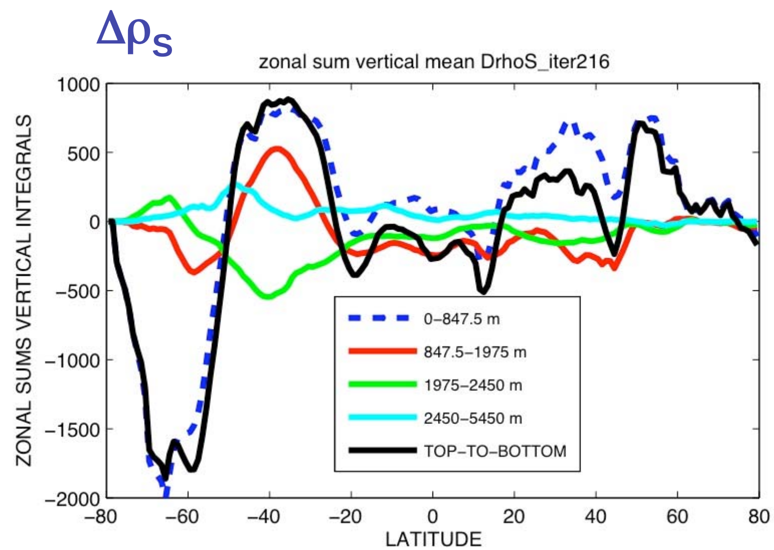
- Approach to remove air-sea flux imbalances:
  - Impose additional constraint in cost function for overall balance in
    - *Evaporation - Precipitation - Runoff*
    - Net heat flux penalized at "observed" *1 W/m*^{2}
  - Adjusted (ECCO) fluxes are balanced as result of adjoint-based optimization within residual errors
  - Misfit in time-varying SSH anomalies are successfully reduced to levels of v2.199 (status quo ante)
  - Balance is achieved over full 1993 to 2004 period (alternative per-year balancing is conceivable).

Application: decadal sea-level patterns (IV)

- Vertical partition in density trends due to
  - trends in temperature T
  - trends in salinity S
  - trends in T, S







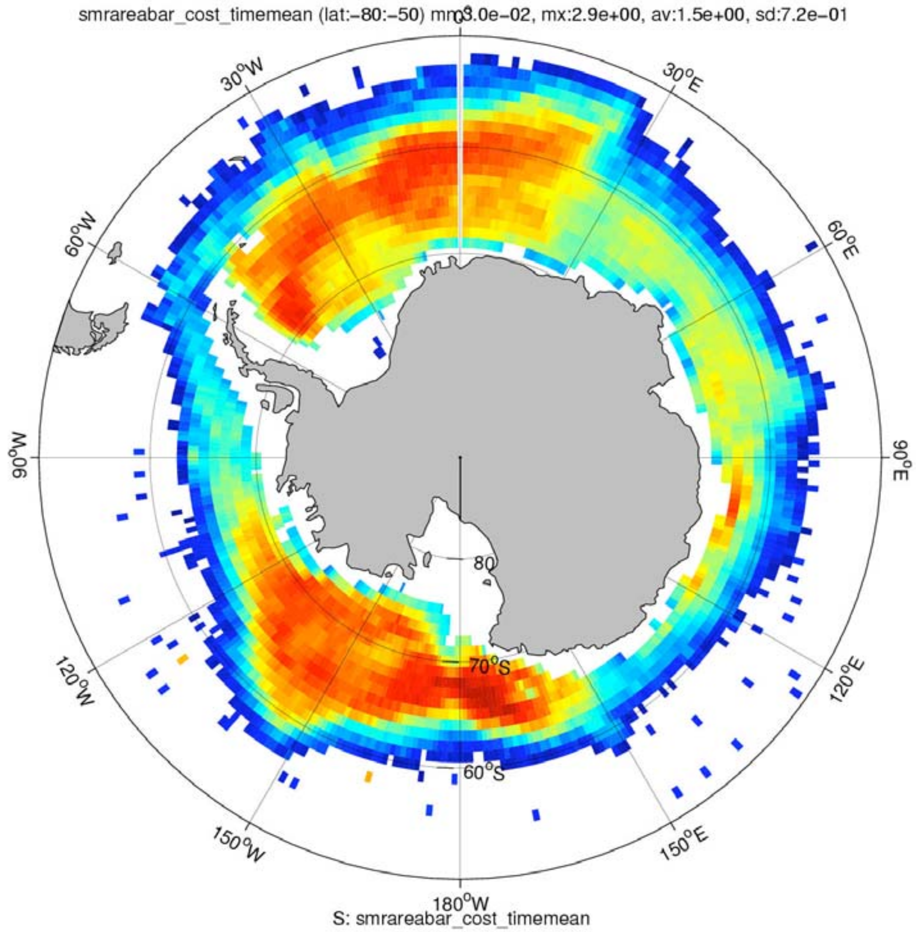
Wunsch et al., 2007: Decadal trends in sea level patterns. (submitted to J. Clim.)

ECCO-GODAE v3.27 (experimental) atmos. boundary layer & sea-ice model

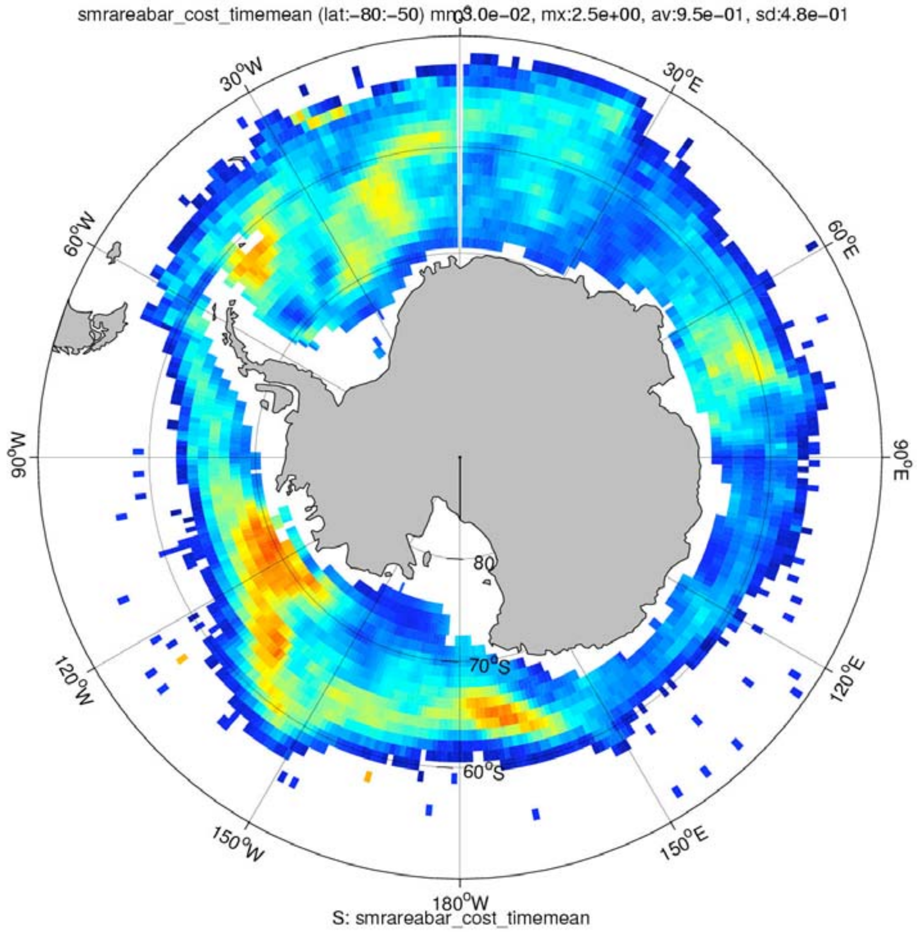
Sea-ice concentration: daily model vs. NSIDC

(National Snow and Ice Data Center)

Iteration 0



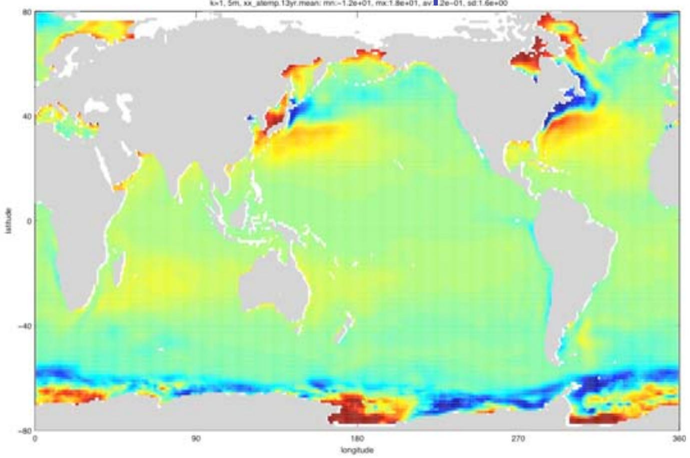
Iteration 27



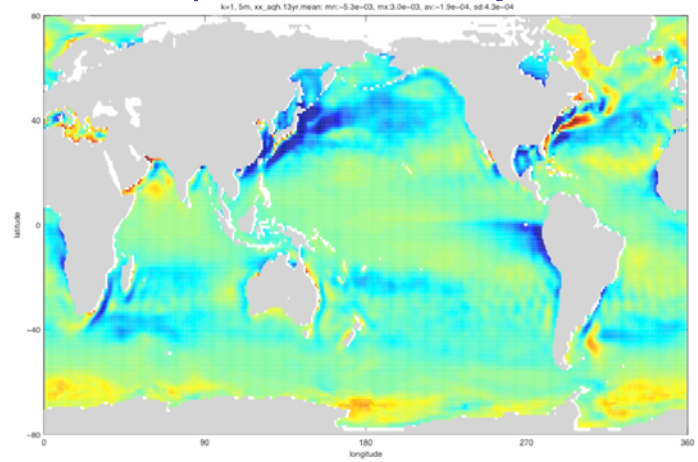
ECCO-GODAE v3.27 (experimental)

Atmospheric state adjustment (controls)

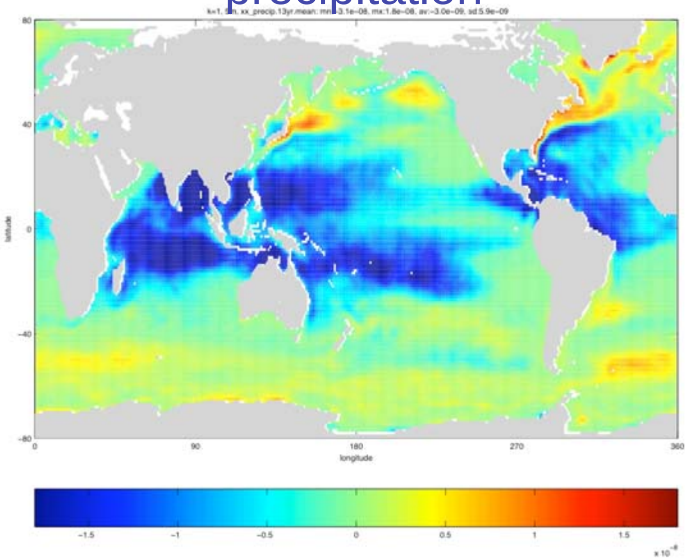
surface air temperature



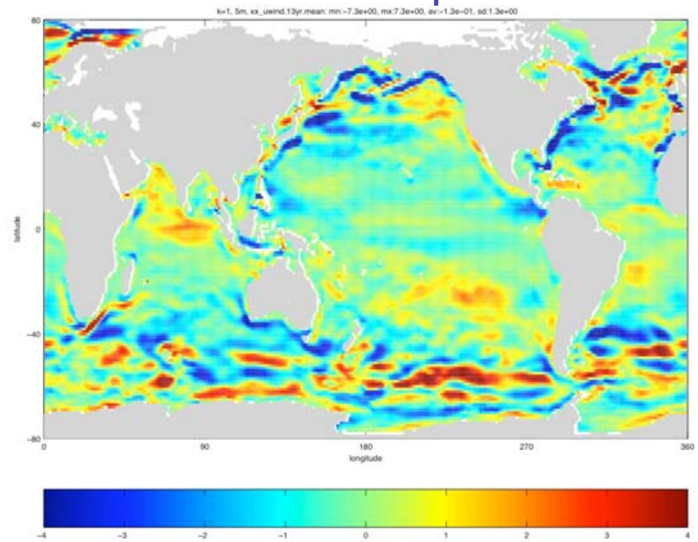
specific humidity



precipitation



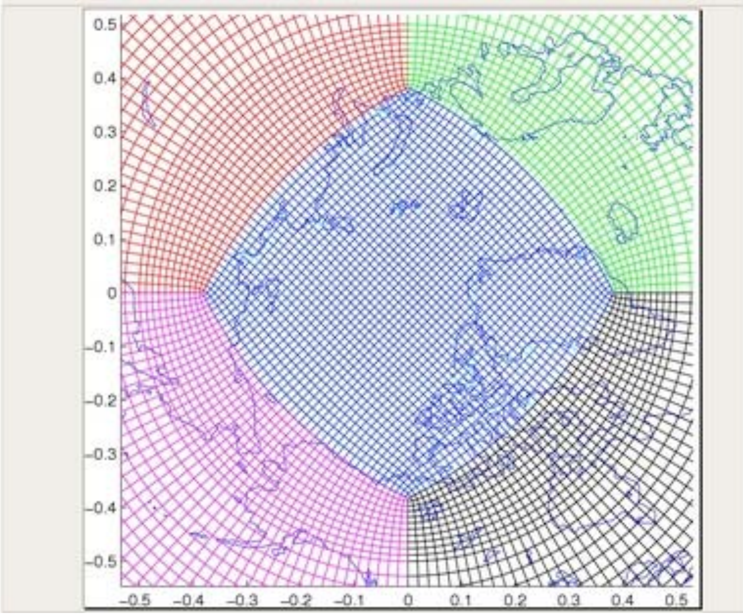
zonal wind speed

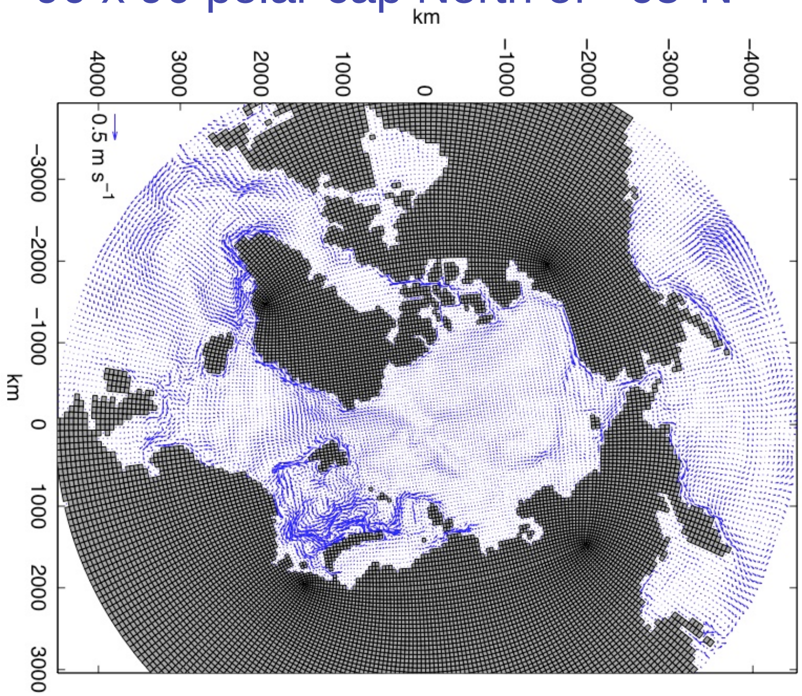


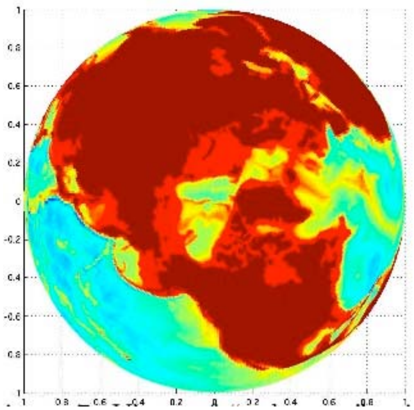
Version 4: SPGrid - a truly global grid (related to cubed-sphere grid)

L(at) **L**(on) **P**(olar) **C**(ap) specifics:
(Hill et al., MWR 2007, submitted)

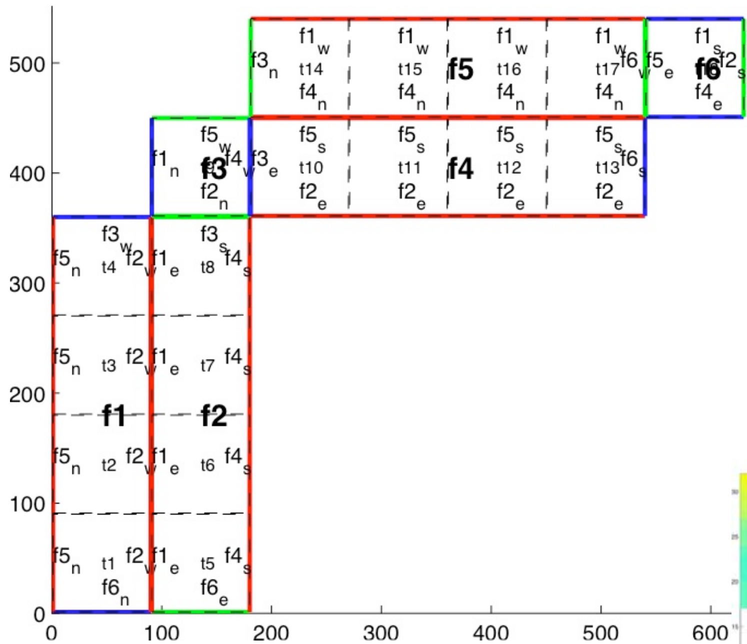
- Topologically equivalent to cubed-sphere
- Nominally 1° (i.e. zonal spacing)
- Lat/Lon between 81°S and 65ºN
- Telescopic from 0.25° to 0.8º between 25ºN/S
- Isotropic to 81ºS
- 90 x 90 polar cap North of ~65ºN

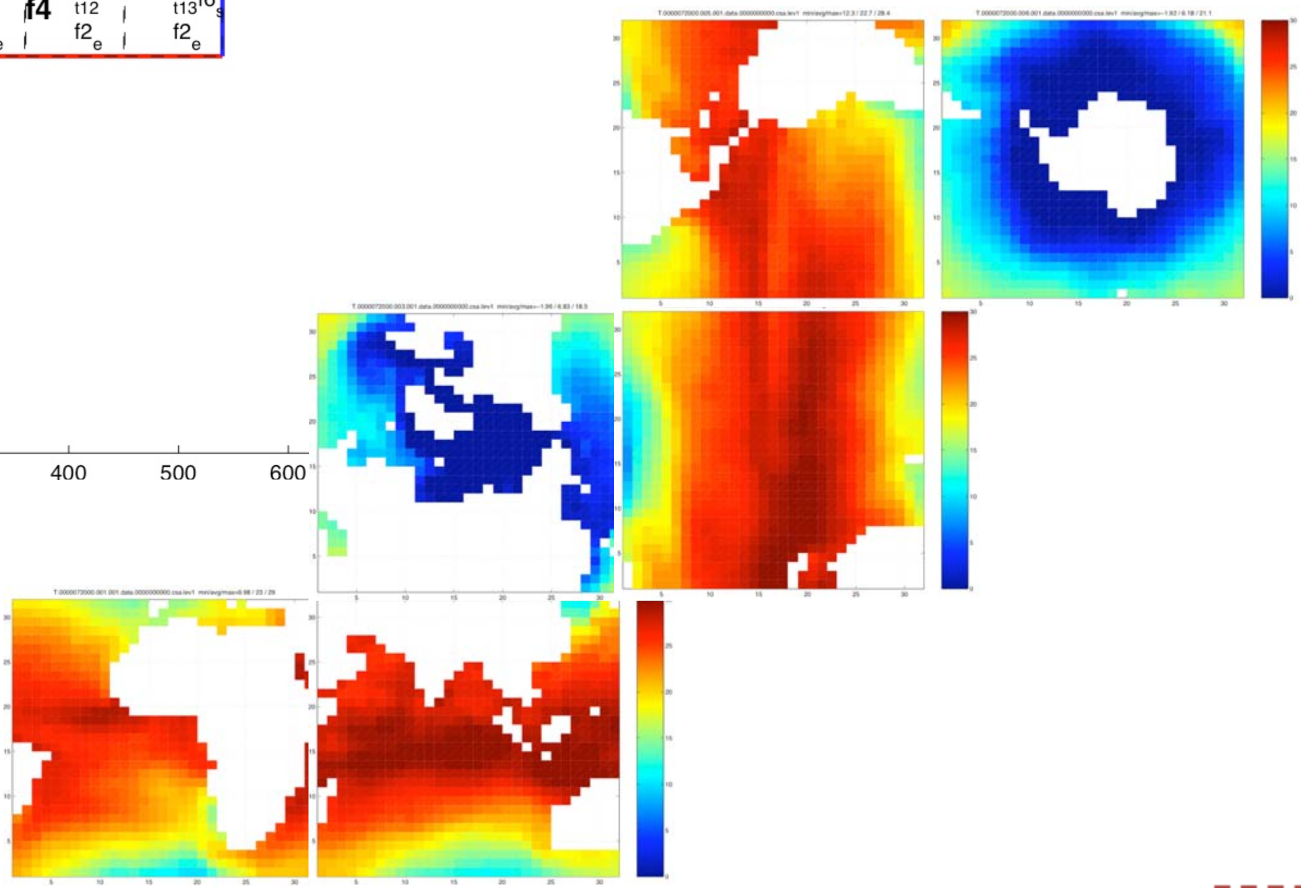






Version 4: matching the adjoint to the cubed-sphere topology





Toward high-resolution state estimation (I)

ECCO2: High-Resolution Global-Ocean and Sea-Ice Data Synthesis @ NASA/Ames

MIT
Marshall,
Campin,
Heimbach, Hill,
Mazloff, Wunsch

JPL
Fu, Kwok, Lee,
Menemenlis,
Zlotnicki

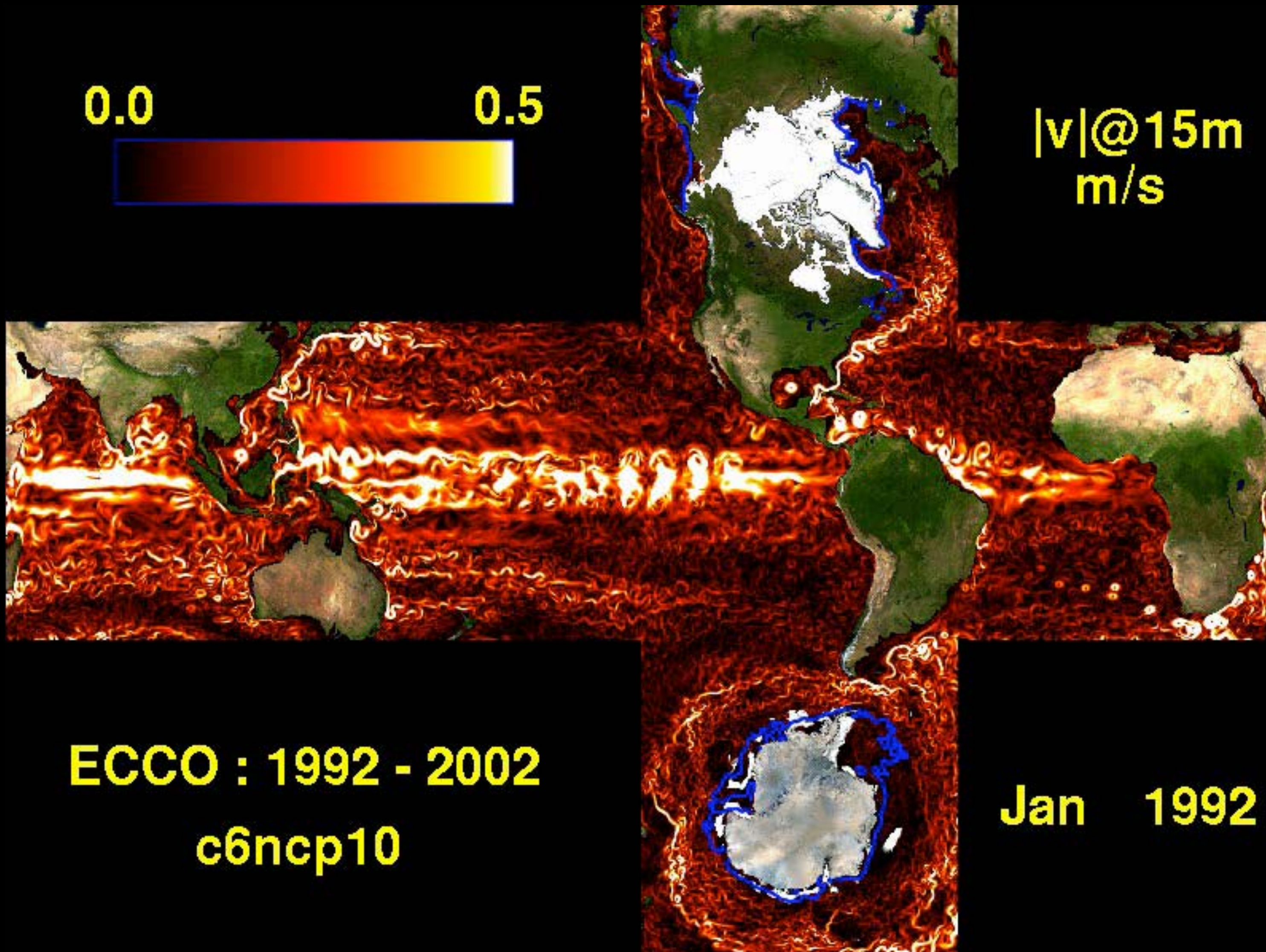
GSFC
Rienecker, Suarez

ARC
Henze, Taft

HARVARD
Tziperman, Zanna

GFDL
Adcroft

ARGONNE
Hovland, Utke



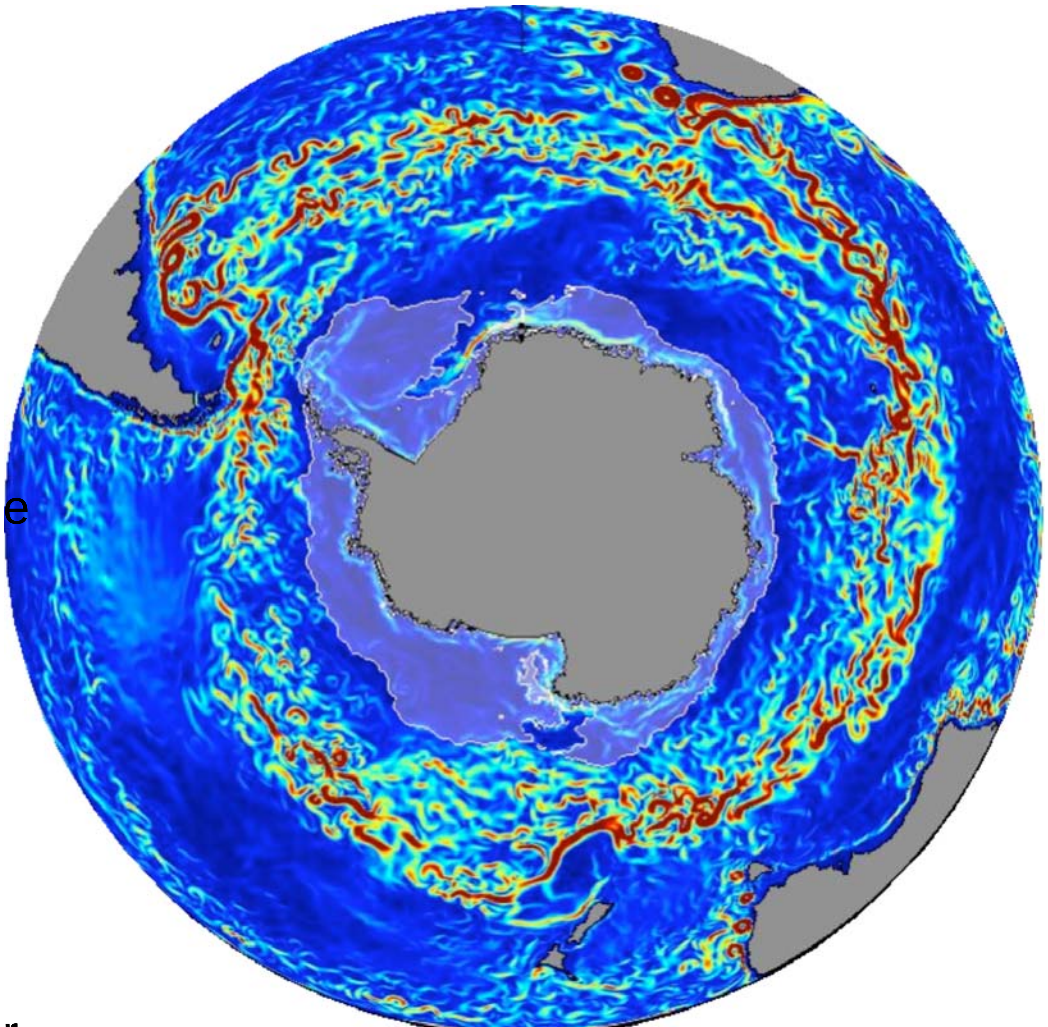
Toward high-resolution state estimation (II)

Eddy permitting state estimation in the Southern Ocean

M. Mazloff (Ph.D. thesis)

- 78^0 South to 24.7^0 South
- $1/6^0$ Horizontal resolution;
- 42 depth levels (partial cells)
- similar setup to ECCO-GODAE
- atmospheric boundary layer scheme
- adjoint generated via AD tool TAF
- sea-ice model
- KPP, GM/Redi parameterizations
- currently optimizing year 2005

- 600 processor adjoint on SDSC's DataStar (IBM SP4) supercomputer

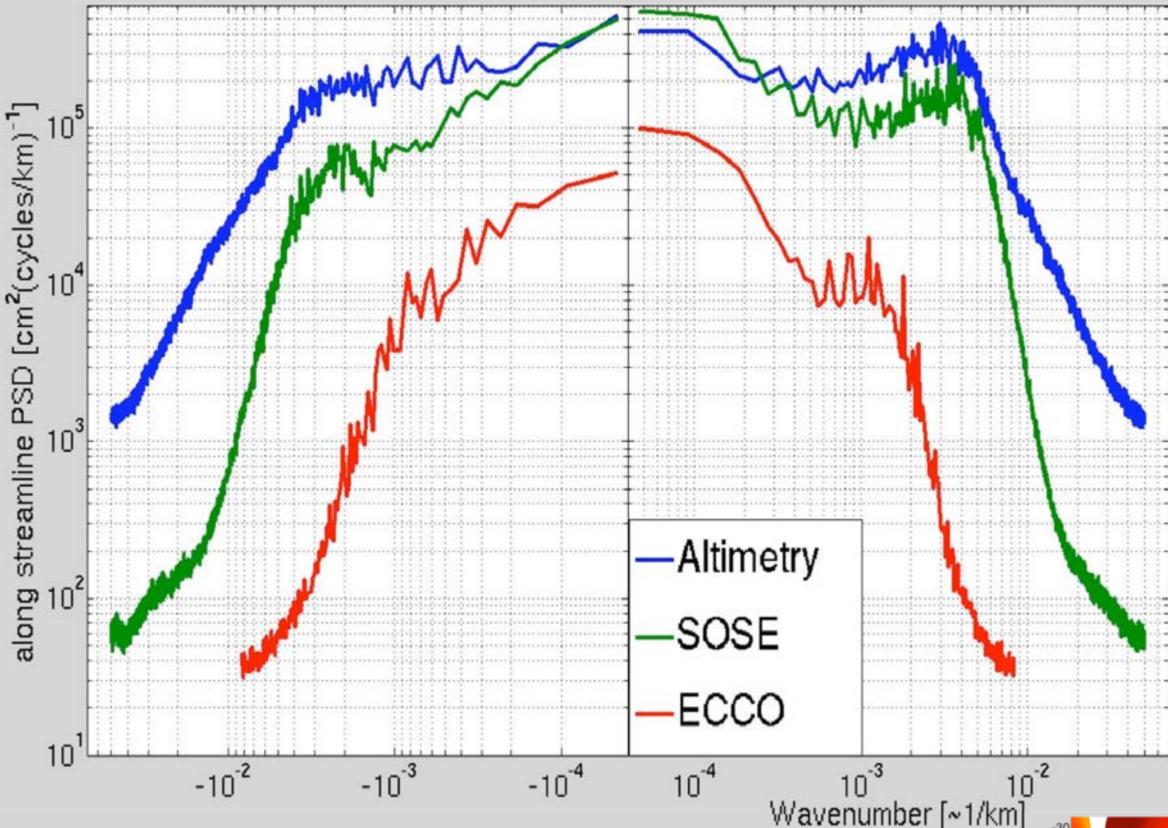


SDSC
SAN DIEGO SUPERCOMPUTER CENTER

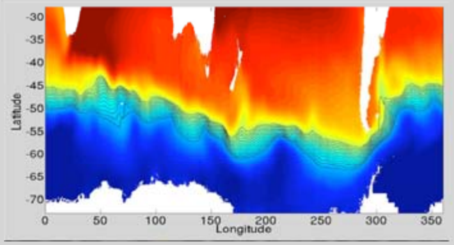
Toward high-resolution state estimation (III)

Eddy permitting state estimation in the Southern Ocean (cont'd)

Along streamline spectra vs. wavenumber (left) and frequency (right)



westward | eastward



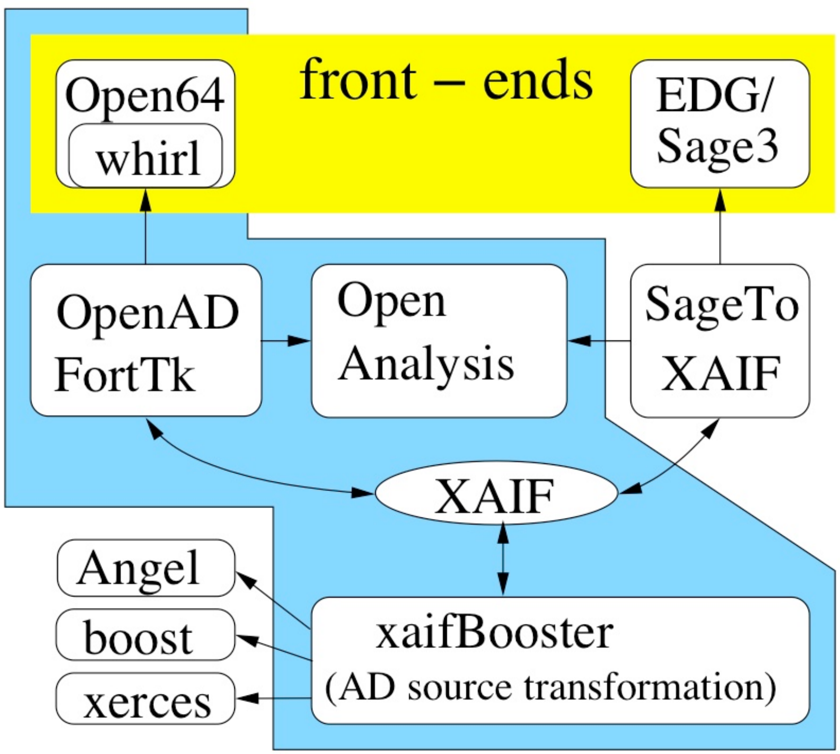
streamlines

M. Mazloff (Ph.D. thesis)

OpenAD: a new open-source automatic differentiation tool

http://www.mcs.anl.gov/OpenAD

@ ***ANL:*** ***J. Utke***, B. Norris, M. Strout, P. Hovland
@ ***Rice:*** N. Tallent, G. Mellor-Crummy, M. Fagan
@ ***MIT:*** P. Heimbach, C. Hill, D. Ozyurt, C. Wunsch
@ ***RWTH:*** ***U. Naumann***



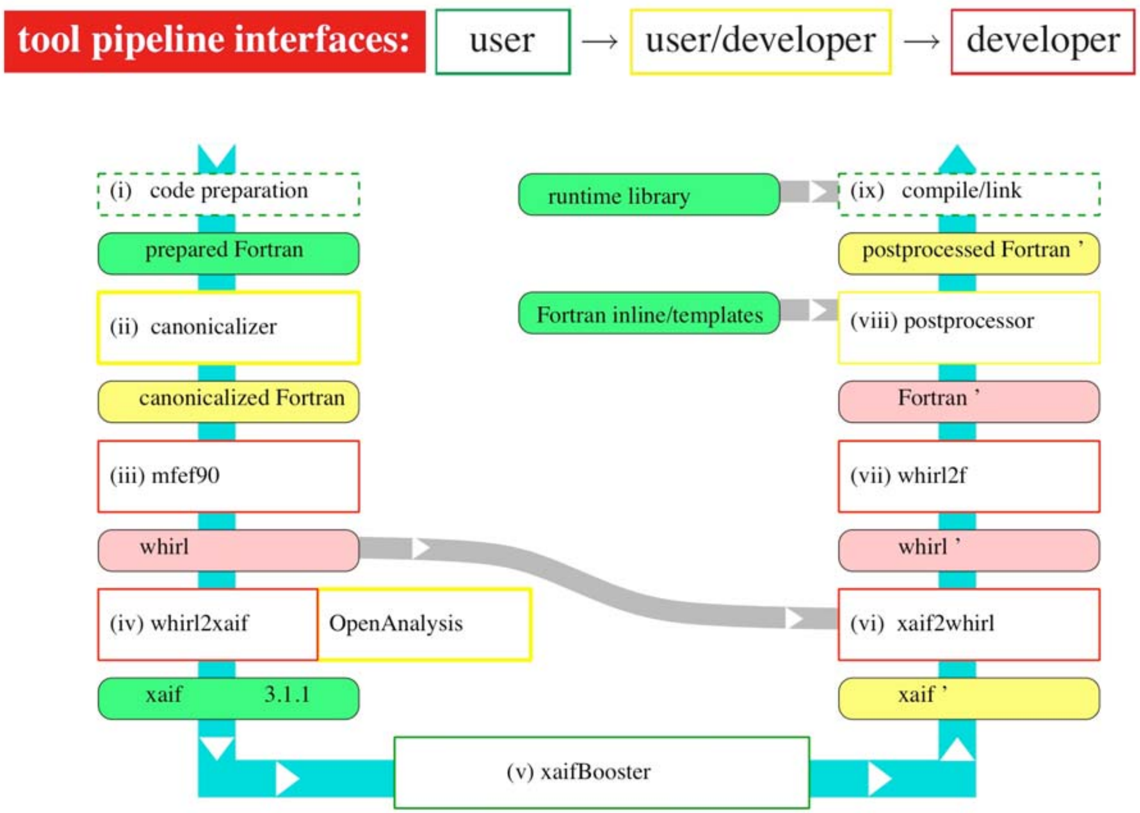
Tool design emphases:

- modularity
- flexibility
- use of open-source components
- new algorithmic approaches

• XML-based language-independent transformation
• basic block preaccumulation
• other optimal elimination methods
• control flow & call graph reversal
• taping & hierarchical checkpointing

OpenAD: a new open-source automatic differentiation tool

http://www.mcs.anl.gov/OpenAD



Utke et al., 2007: submitted to ACM Transactions on Math. Software (TOMS).

Open64:

(code parsing)

- lexical/syntactic/semantic analysis
- canonicalizer
- intermediate representation

OpenAnalysis:

(static code analysis)

- build call / control flow graphs
- code analysis, activity, side-effects

whirl2xaif / xaif2whirl:

(representing the numerical core)

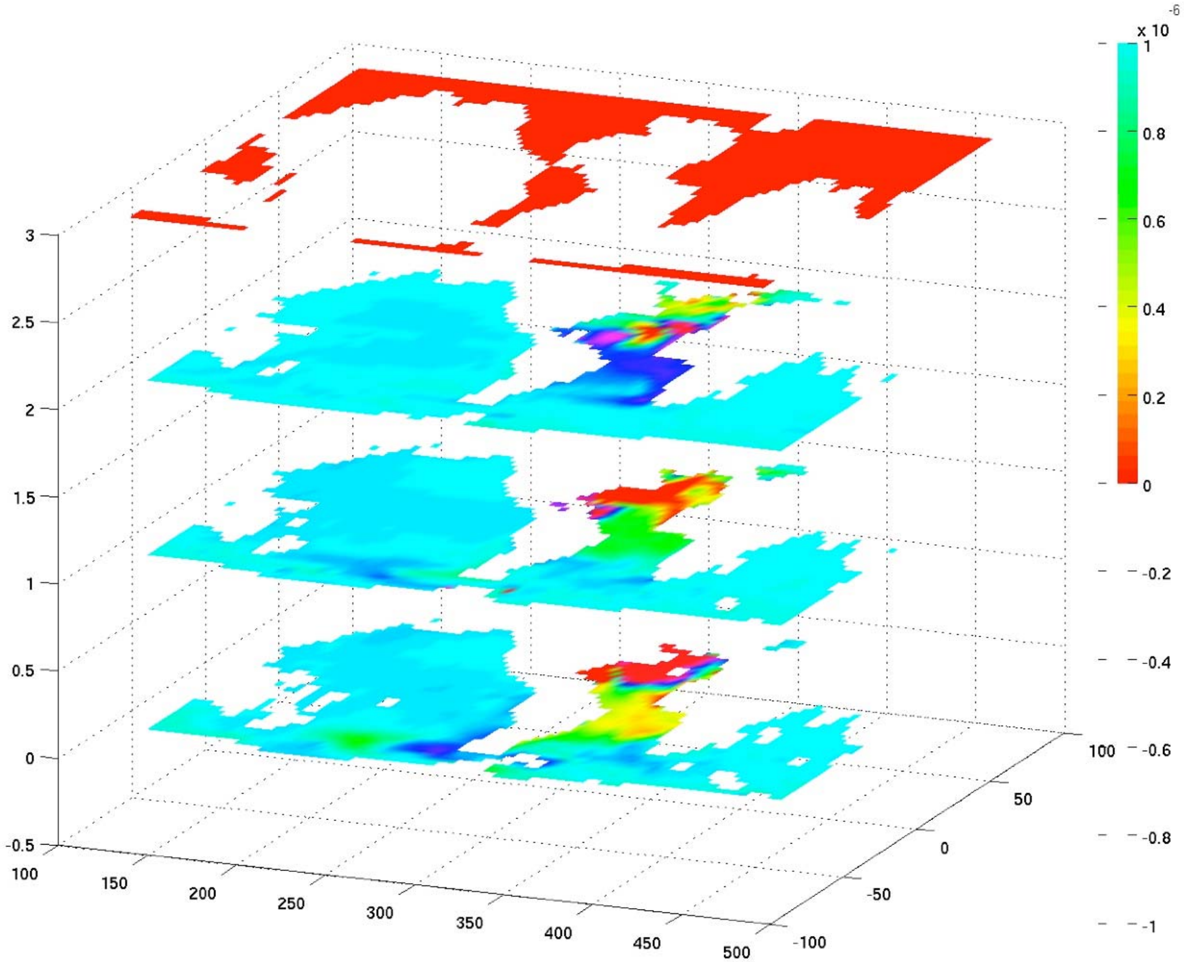
- representation in language-independent XAIF format

xaifBooster:

(transforming the numerical core)

- apply differentiation algorithms

Atlantic meridional heat transport: 5 year sensitivities at 4º resolution (OpenAD)

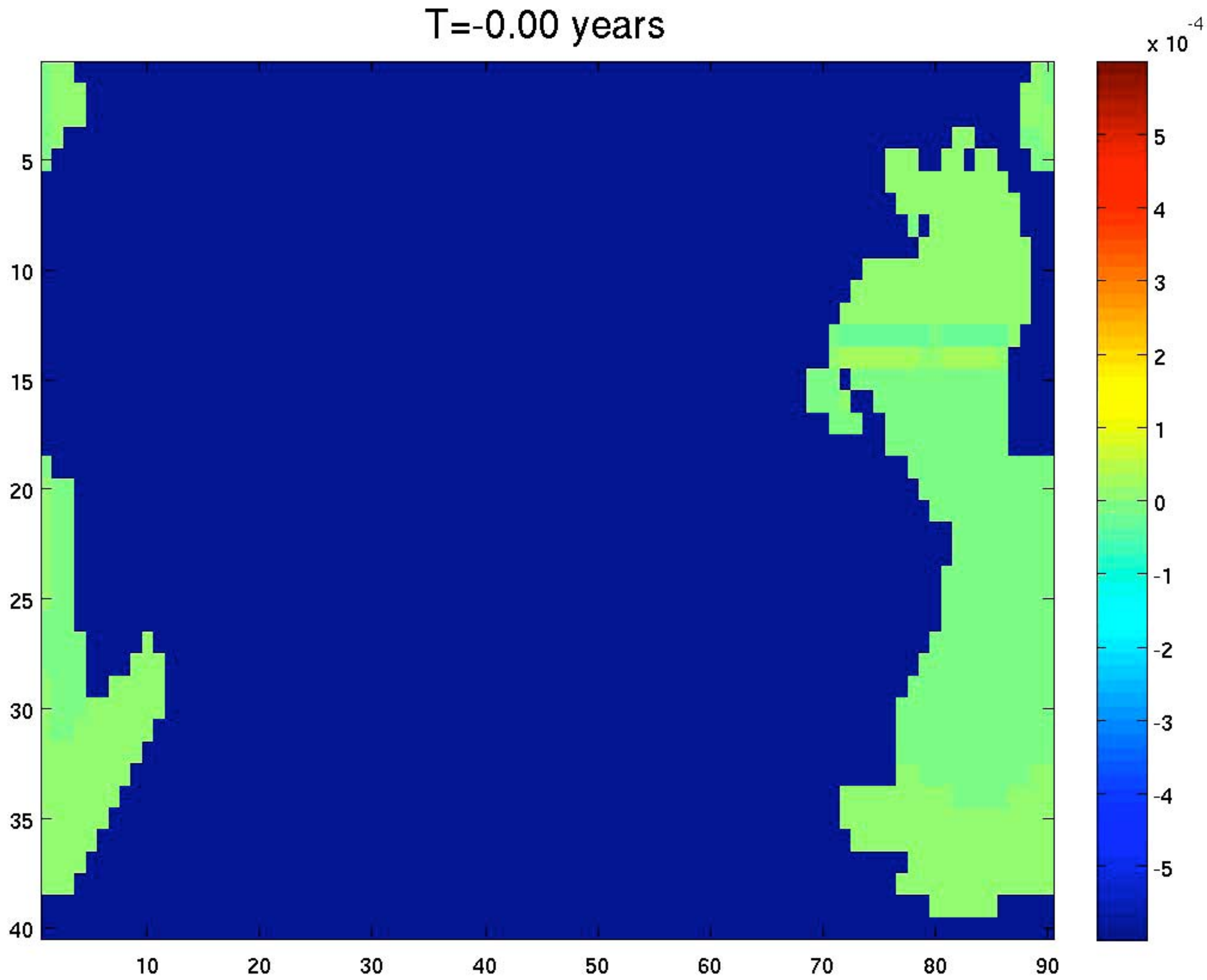


First ***MITgcm*** application using ***OpenAD***, and with implemented checkpointing at the time-stepping level.

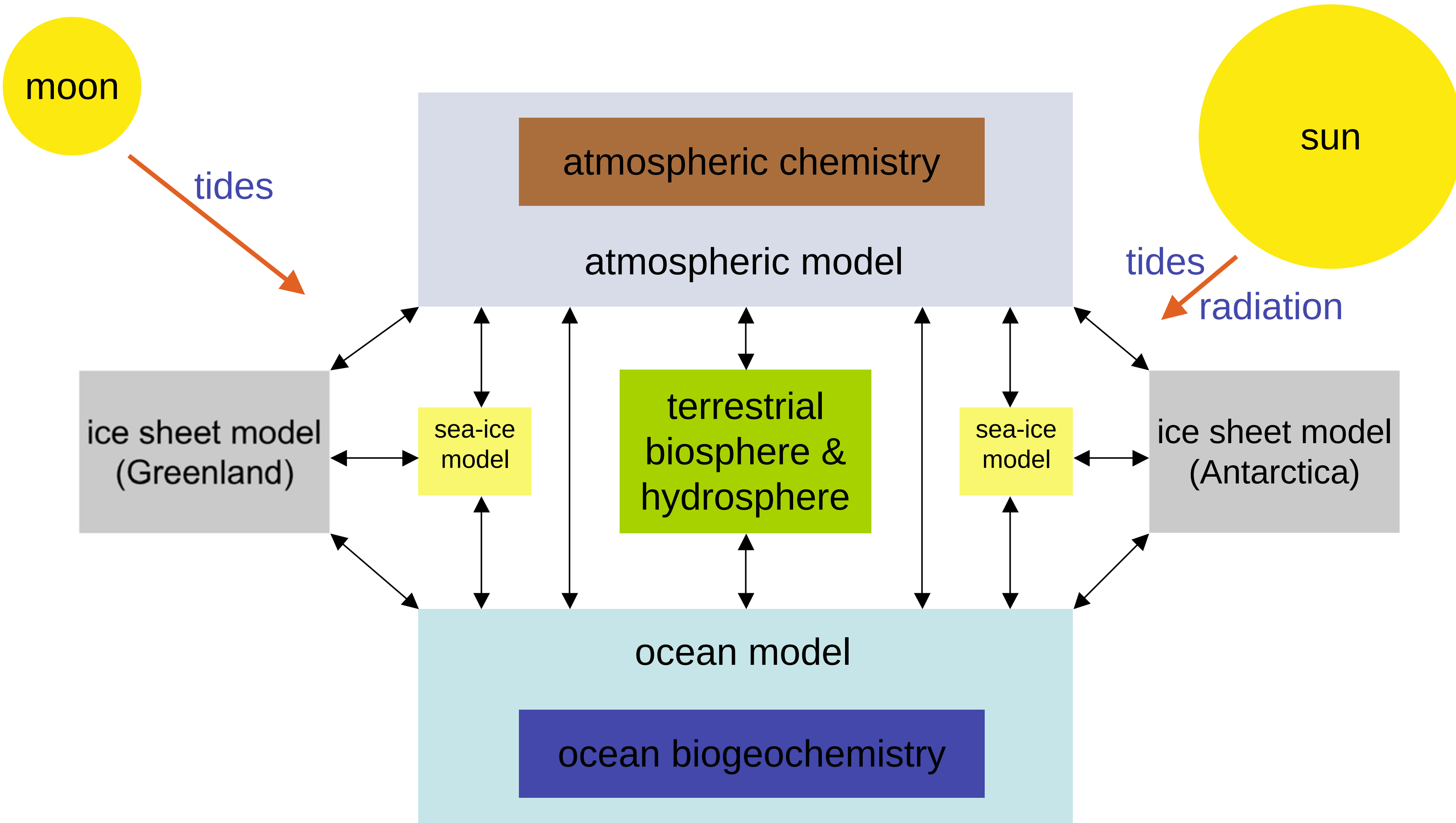
Extend adjoint integration of heat flux sensitivities backward in time (here at coarser resolution).

Confirms role of propagating waves (Rossby waves, Kelvin waves) over these time scales in fast signal propagation over long distances.

Atlantic meridional heat transport: 10 year sensitivities at 4° resolution (OpenAD)



Toward a coupled adjoint Earth System Modeling System



Outlook (I)

- Observations and uncertainties
  - new types (e.g. acoustic tomography, time-varying GRACE)
  - determining scales, errors, and covariances
  - sparsity of observations
  - satellites: maintaining long-term climate-relevant missions (similar to weather satellites, but look at 20- to 50-year horizon)
- Model and adjoint
  - high-resolution adjoint and exponential sensitivity growth (linked to Lyapunov exponent, predictability horizon, …)
  - representation error due to model vs. obs. scales mismatch
  - model error, and model error covariances
  - long-term state estimation (100 to 1000 year time scales)
  - coupled atmosphere-ocean problem (fast vs. slow timescales)
  - scientific interpretations of remaining misfits (inconsistencies)

Outlook (II)

- Optimization
  - is a gradient-descent method the best method?
  - are there other/better methods out there for large-scale optimization that we should know?
- Sustaining the effort
  - transfer from science to operational community
  - sustained (and increasing) compute power required
  - Who takes on the challenge of maintaining climate-relevant observational record (in particular satellite)?

state estimation remains essential:
Ability to synthesize, in an optimal manner, all available observations and best known physics/dynamics (a model) to derive a full state of the ocean that is consistent with known physics and observations, and yields closed budgets to enable analysis of the nature and causes of variability and change.

An early vision, ca. 1982:

Acoustic Tomography and Other Answers

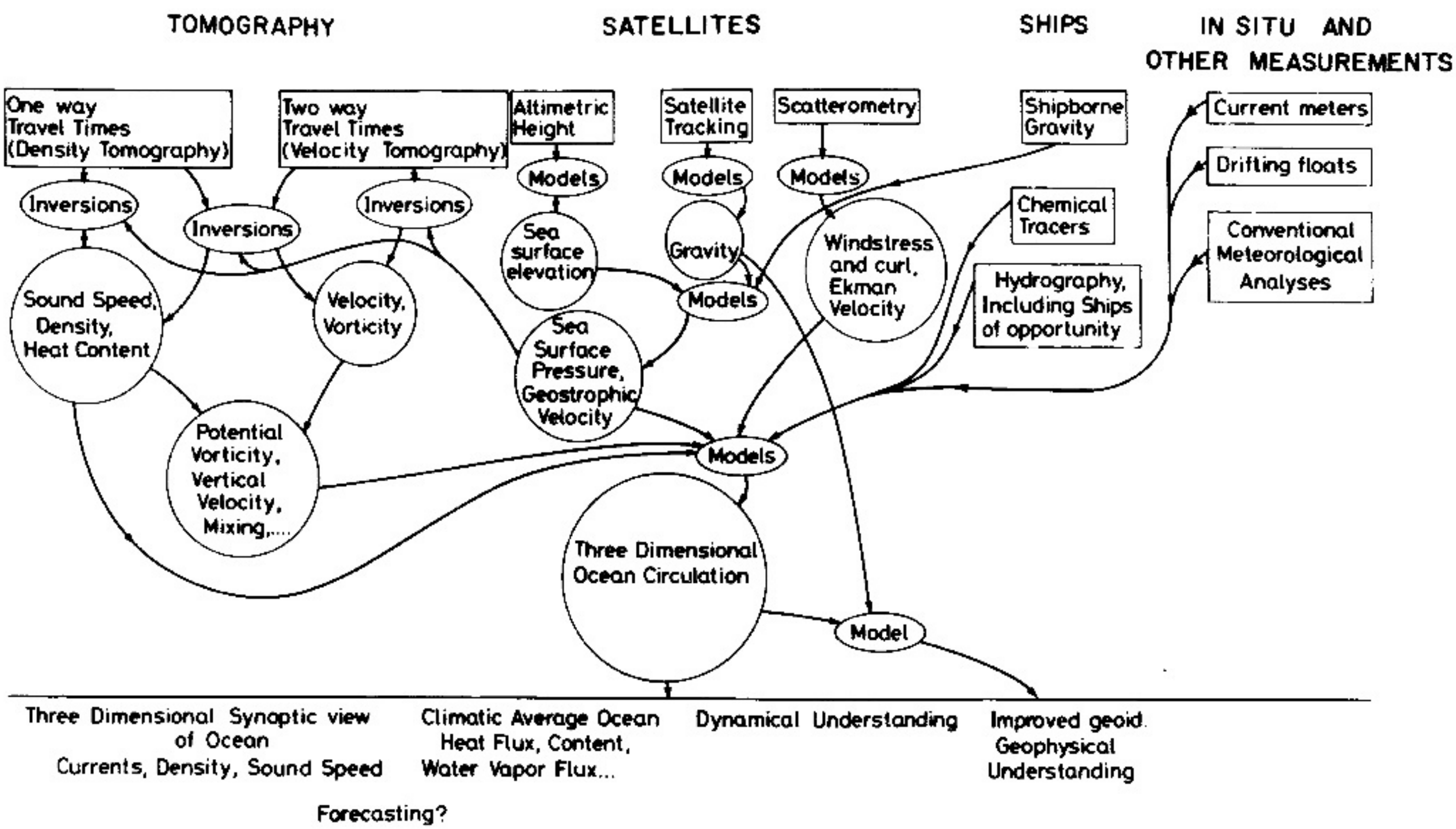


Figure 26. All measurements and models of the ocean can be interconnected to provide global estimates of the state of the three-dimensional ocean. Some side benefits accrue — e.g. improved estimates of the earth's gravity field.

Taken from: ***C. Wunsch***, in *"A Celebration in Geophysics and Oceanography 1982. In Honor of Walter Munk on his 65th birthday."*
C. Garrett and C. Wunsch, Eds., SIO Reference Series 84-5, March 1984